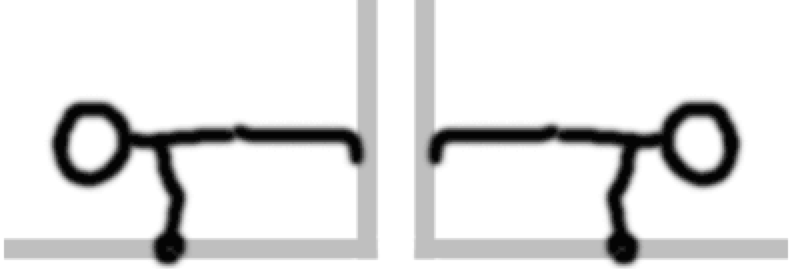
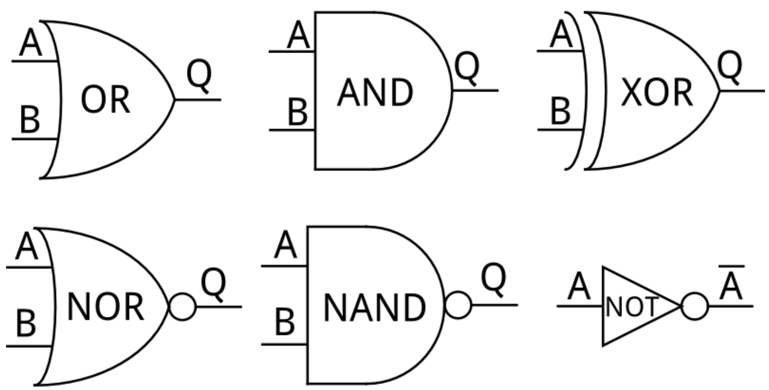
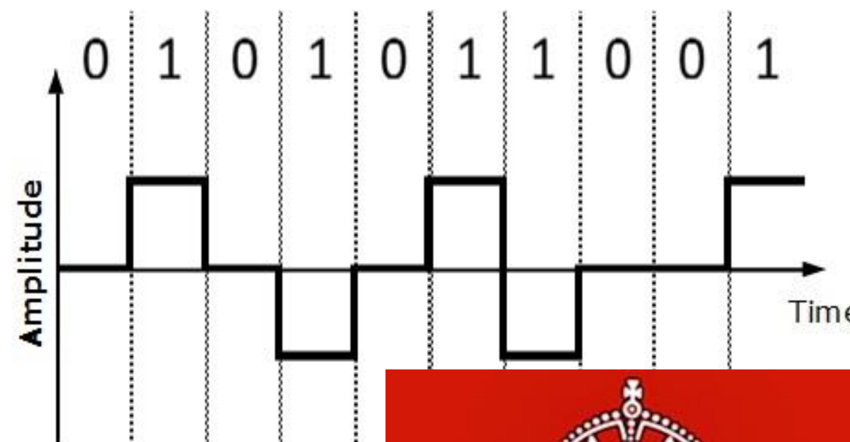


Proiectare Logica

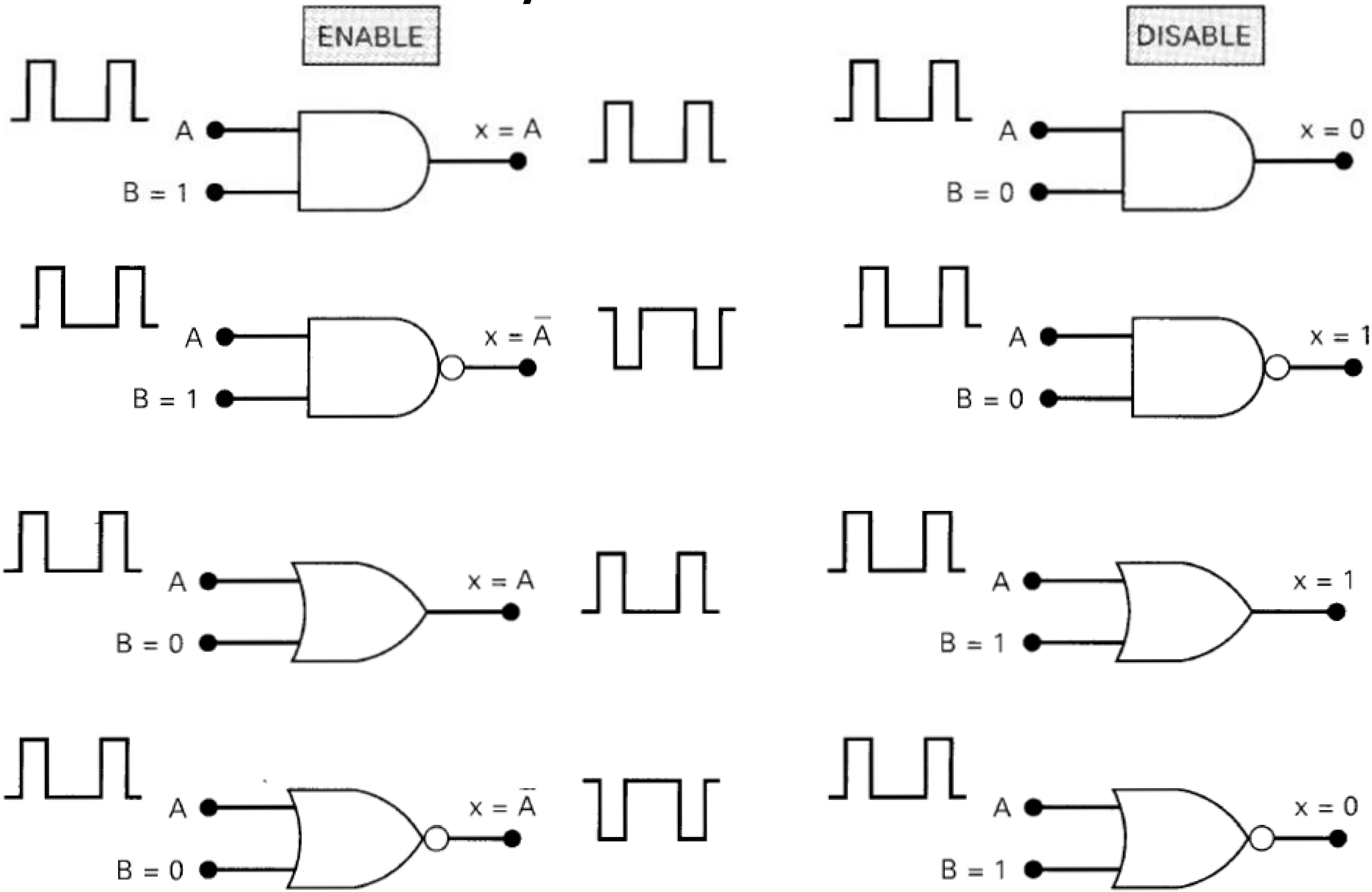
Digital Logic Design

Curs 04

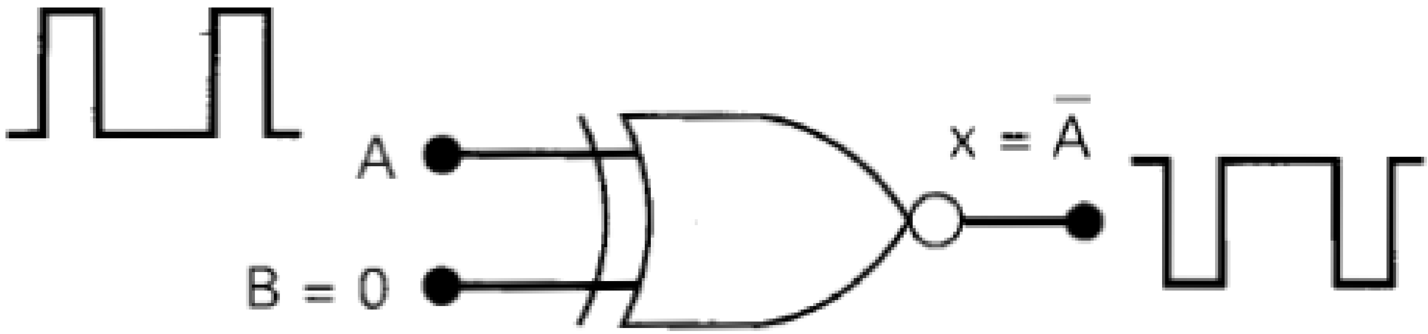


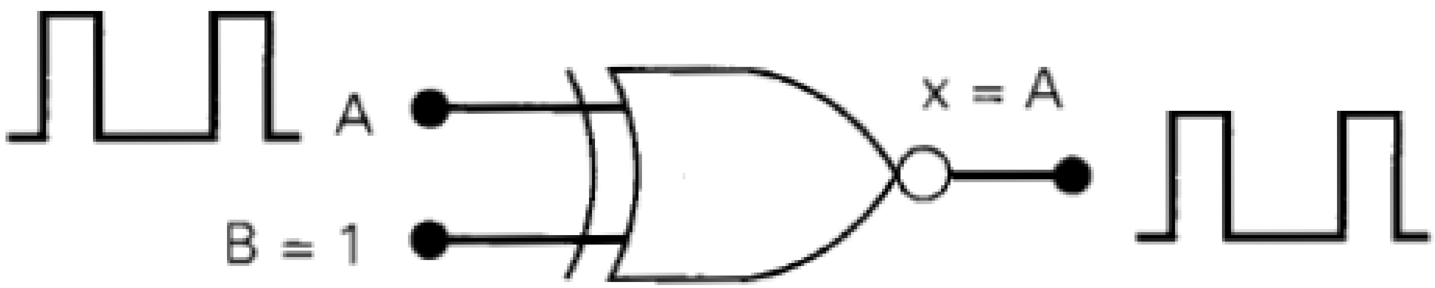


ENABLE/DISABLE Circuits

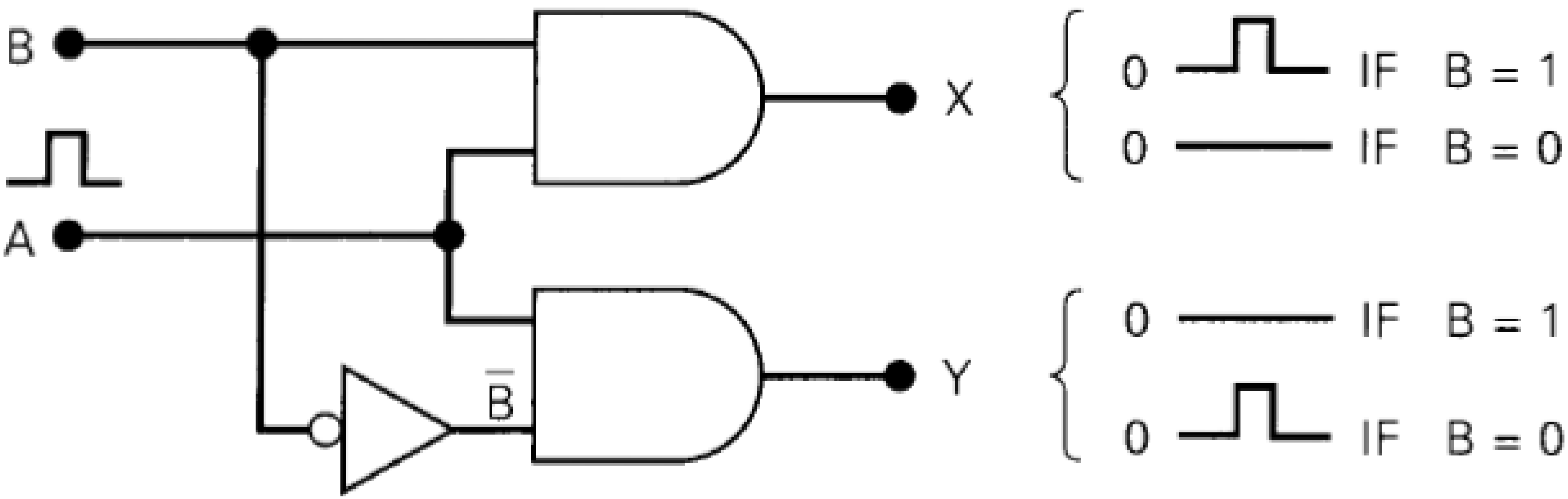


ENABLE/DISABLE Circuits – XNOR case

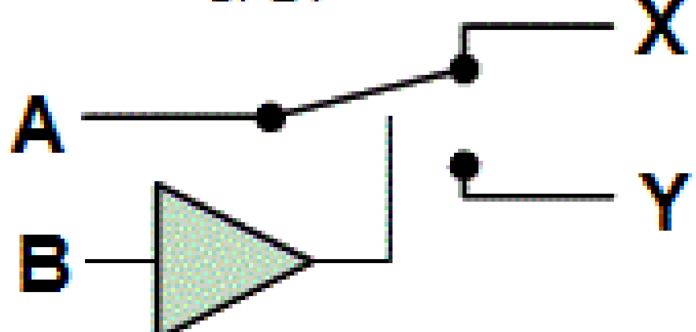




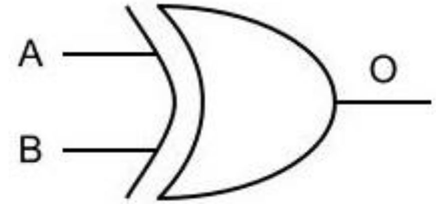
Circuitul de dirijare a pulsului–pulse-steering circuit



Single Pole Double Throw Switch
SPDT



Functia (operatorul) XOR



A	B	O
0	0	0
0	1	1
1	0	1
1	1	0

- Functia **XOR**, sau **operatorul Modulo 2**, **⊕**, este definita prin:

$$A \oplus B = \bar{A}B + A\bar{B}$$

- Din principiul dualitatii ➔:

$$\overline{A \oplus B} = (A + \bar{B}) \cdot (\bar{A} + B)$$

- Alte proprietati:

$$A \oplus A = 0 \qquad A \oplus \bar{A} = 1$$
$$A \oplus 1 = \bar{A} \qquad A \oplus 0 = A$$
$$A + B = A \oplus B \oplus AB$$
$$A + B = A \oplus B \text{ if } AB = 0$$

$$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$
$$A \oplus B = B \oplus A$$
$$A(B \oplus C) = AB \oplus AC$$

If $A \oplus B = C$ then $A = B \oplus C$, $B = A \oplus C$ and $A \oplus B \oplus C = 0$

Curs 4

Exemplu bilet partial

- 1. Scrieti numarul 755_8 in baza 10.
- 2. Care sunt primele 7 cifre dupa virgula ale numarului 0.458_{10} in baza 8.
- 3. Scrieti tabela de adevar a functiei F(ABCD)=A.B'.C+B.D+A.B.C+A.C'. Gasiti mintermenii acestei functii si scrieti FCD.
- 4. Functia f cu patru variabile are maxtermenii (2,1,10,12,4,11,7,8). Minimizati aceasta functie folosind diagramele Veitch.
- 5. Gasiti reprezentarea in binary64 a numarului -12.062510 (bit: semn, exponent si mantisa) folosind primul algoritm.

Seturi complete

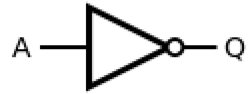
- **Orice functie booleana poate fi exprimata folosind numai operatorii AND si NOT.**

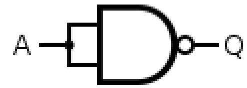
In aplicatiile electronice folosim porti AND si INVERSOARE pentru operatiile AND si NOT.

- **Poarta NAND, derivata din poarta NOT si un AND poate fi utilizata la implementarea oricarei funtii logice (vezi pagina urmatoare).**

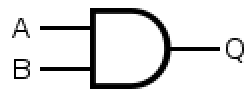
Implementari folosind NAND-uri

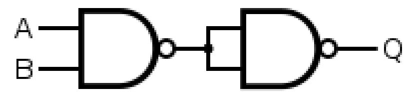
- NOT



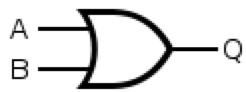


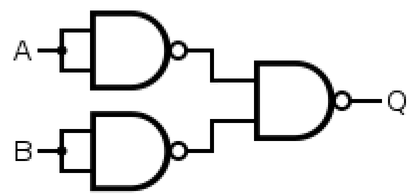
- AND



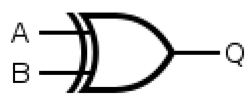


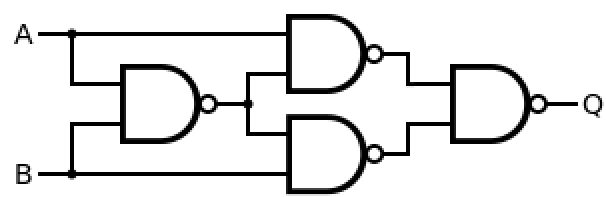
- OR





- XOR





Forme canonice

$\mathbf{0 \cdot A = 0; 0 + A = A}$

dispar termenii pentru care $\boldsymbol{a_i = 0}$

Suma de produse

- Forma canonica disjunctiva FCD:
  - $f(A, B, C) = a_0A'B'C' + a_1A'B'C + a_2A'BC' + \cdots + a_7ABC = \sum_{i=0}^{2^3-1} a_i m_i$, unde m_i se numeste **mintermenul** *i*.

$\mathbf{1 + A = 1; 1 \cdot A = A}$

dispar factorii pentru care $\boldsymbol{b_i = 1}$

Produs de sume

- Forma canonica conjunctiva FCC:
  - *g(A, B, C)* =$(b_0 + A + B + C) \cdot (b_1 + A + B + C') + \cdots + (b_7 + A' + B' + C') = \prod_{i=1}^{2^3-1} M_i$, unde M_i se numeste **maxtermenul** *i*.

$\boldsymbol{b_0} \rightarrow \boldsymbol{A + B + C = (A'B'C')'}$

$\boldsymbol{b_1} \rightarrow \boldsymbol{A + B + C' = (A'B'C)'}$

← De Morgan

Forme canonice: exemplu

- Fie functia *f* definita de tabelul:

i	A	B	C	f	mintermeni (m_i)	Maxtermeni (M_i)
0	0	0	0	**1**	**A'B'C'**	A+B+C
1	0	0	1	**0**	A'B'C	**A+B+C'**
2	0	1	0	**0**	A'BC'	**A+B'+C**
3	0	1	1	**1**	**A'BC**	A+B'+C'
4	1	0	0	**1**	**AB'C'**	A'+B+C
5	1	0	1	**0**	AB'C	**A'+B+C'**
6	1	1	0	**1**	**ABC'**	A'+B'+C
7	1	1	1	**1**	**ABC**	A'+B'+C'

FCD: $f = A'B'C' + A'BC + AB'C' + ABC' + ABC = \sum m(0,3,4,6,7)$

FCC: $f = (A+B+C')(A+B'+C)(A'+B+C') = \prod M(1,2,5)$

min-max termenii

Determinati min-max termenii functiei din tabelul urmator:

Tabela de adevar

A	B	C	F (ABC)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Avem mintermenii:

$m_1 = A'B'C, m_3 = A'BC, m_4 = AB'C',$
$m_6 = ABC'$

Adica $F = A'B'C + A'BC + AB'C' + ABC'$

Maxtermenii sunt:

$M_0 = A + B + C, M_2 = A + B' + C,$
$M_5 = A' + B + C'$ si $M_7 = A' + B' + C'$

min-max termenii : demonstratie

Cine este F'? Tabela lui F' este urmatoarea:

$$F' = A'B'C' + A'BC' + AB'C + ABC$$

Negam expresia de mai sus ($F = F''$) si din formulele DeMorgan gasim:

FCC:

$$F = (A + B + C)(A + B' + C)(A' + B + C')(A' + B' + C')$$

Factorii sunt chiar maxtermenii

$M_0 = A + B + C, M_2 = A + B' + C,$
$M_5 = A' + B + C'$ si $M_7 = A' + B' + C'$

FCD:

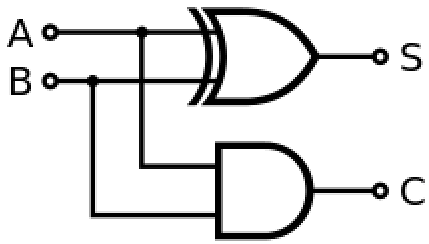
$$F = A'B'C + A'BC + AB'C' + ABC'$$

Tabela de adevar

A	B	C	F (ABC)	F'(ABC)
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	0	1
1	1	0	1	0
1	1	1	0	1

Sumatorul si semisumatorul

- Semisumatorul (HA – Half Adder)

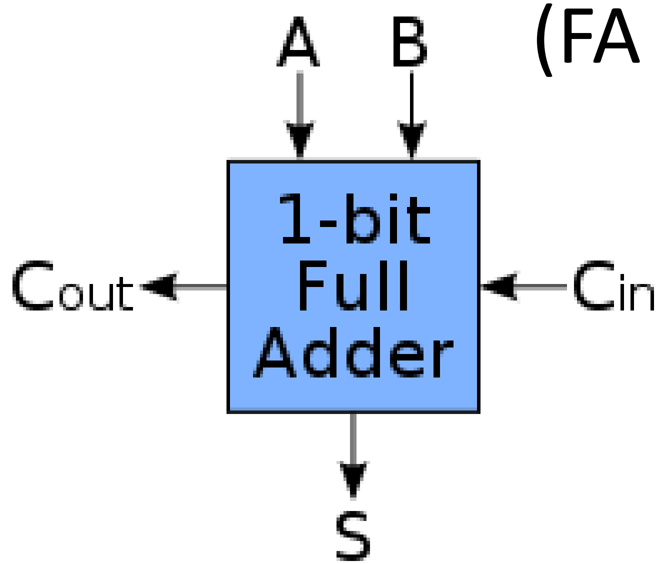


$$S = A \oplus B$$
$$C = A \cdot B$$

Inputs		Outputs	
A	*B*	*C*	*S*
0	0	0	0
1	0	0	1
0	1	0	1
1	1	1	0

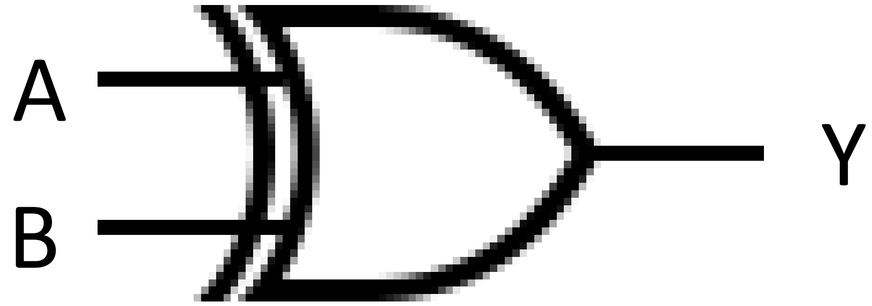
Inputs			Outputs	
A	*B*	C_{in}	C_{out}	*S*
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

- Sumatorul complet (FA – Full Adder)



XOR as (non)inverter gate

- $A = 0 \rightarrow Y = B$
- $A = 1 \rightarrow Y = NOT\ B$



Putem privi semnalul (variabila) A ca pe un semnal de control care permite transmiterea semnalului B inversat sau nu.

A	B	$Y = A\ XOR\ B$
0	0	0
0	1	1
1	0	1
1	1	0

Sumatorul complet - Suma

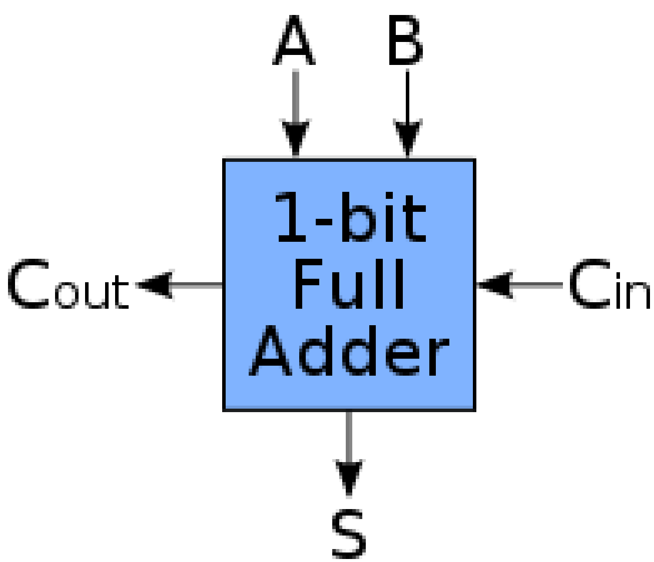
Observam ca:

$$A = 1 \rightarrow S = \overline{B \oplus C_{in}}$$

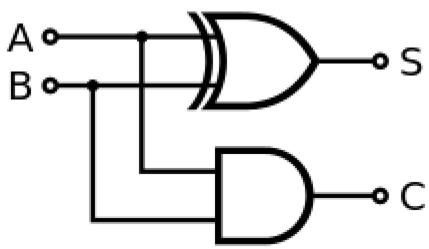
$$A = 0 \rightarrow S = B \oplus C_{in}$$

$$\rightarrow S = (B \oplus C_{in}) \oplus A =$$
$$= (A \oplus B) \oplus C_{in}$$

Inputs			Outputs	
A	B	C_{in}	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



Semisumatorul



$$S = A \oplus B$$
$$C = A \cdot B$$

Sumatorul – Carry out

Observam ca:

$$A = 1 \rightarrow C_{out} = B + C_{in}$$
$$A = 0 \rightarrow C_{out} = B \cdot C_{in}$$

$$\rightarrow C_{out} = \bar{A}BC_{in} + A(B + C_{in})$$

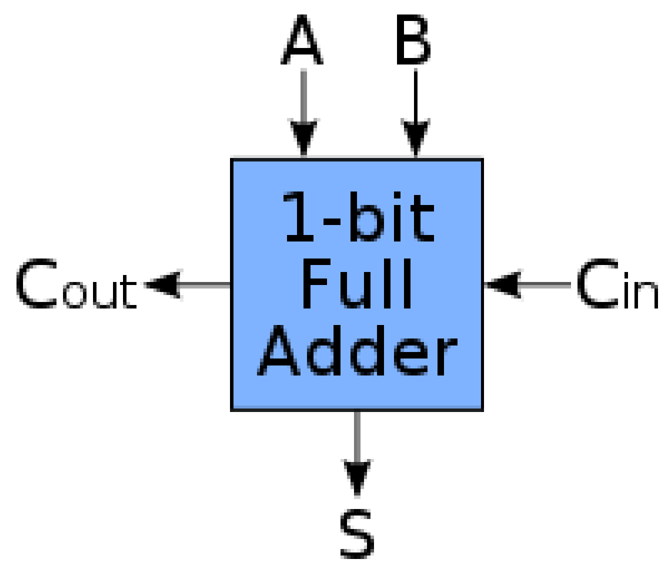
De demonstrat ≡

$$C_{out} = AB + C_{in}(A \oplus B)$$

De demonstrat ≡

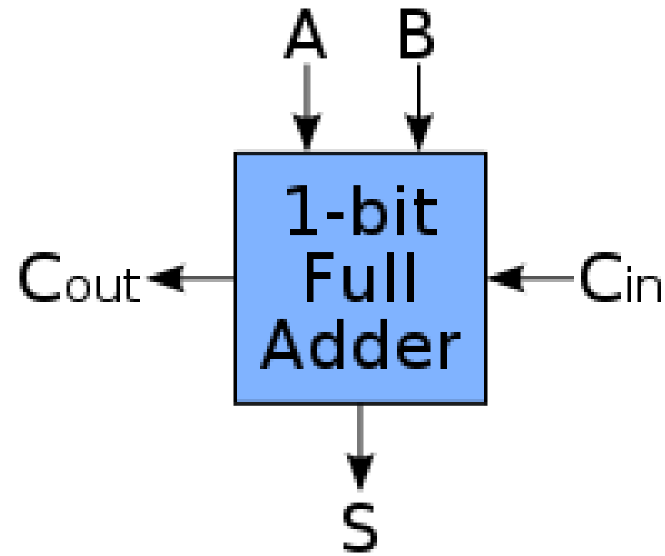
$$C_{out} = AB + C_{in}(A + B)$$

Inputs			Outputs	
A	B	C_{in}	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

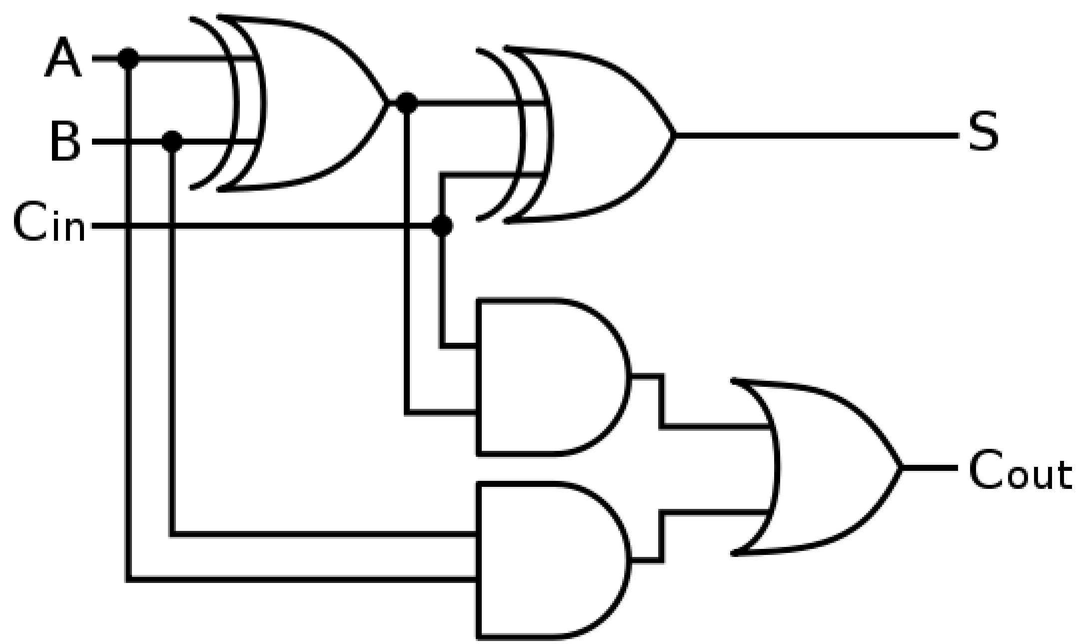


Sumatorul complet

$$S = (A \oplus B) \oplus C_{in}$$
$$C_{out} = AB + C_{in}(A \oplus B)$$



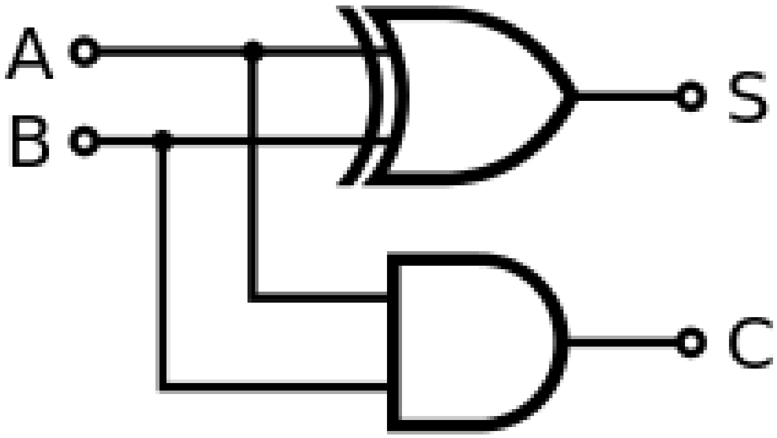
Inputs			Outputs	
A	B	C_{in}	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



Semi-sumatorul

$$S = A \oplus B$$
$$C = A \cdot B$$

Inputs		Outputs	
A	*B*	*C*	*S*
0	0	0	0
1	0	0	1
0	1	0	1
1	1	1	0



Tema

Sa se arate ca:

1. $(B \oplus C) \oplus A = (A \oplus B) \oplus C$
2. $\bar{A}BC + A(B + C) = AB + C(A \oplus B)$
3. $AB + C(A \oplus B) = AB + C(A + B)$

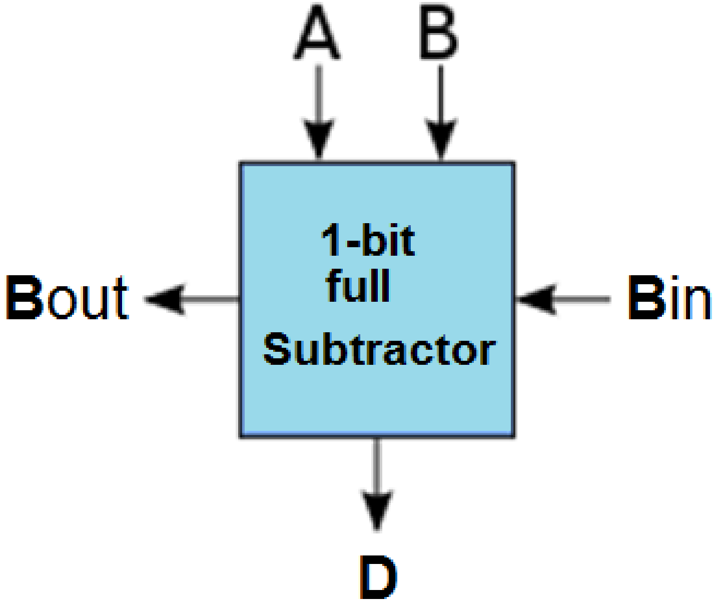
folosind:

a) tabele de adevar

b) diagrame Venn

Scazatorul complet

- Face operatie de scadere $A - B$ pe 1 bit cu variabila de imprumut la intrare B_{in} si variabila de imprumut la iesire B_{out}.

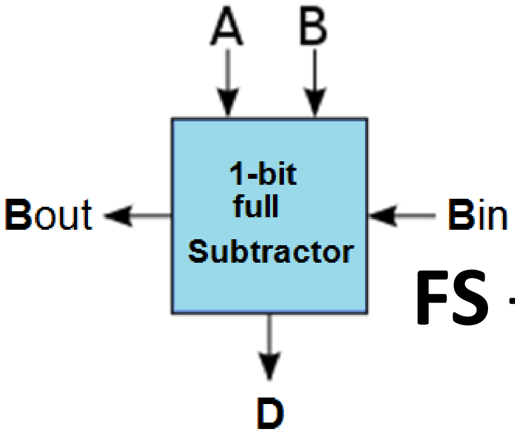


Inputs			Outputs	
A	*B*	B_{in}	D	B_{out}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$$D = (A \oplus B) \oplus B_{in}$$

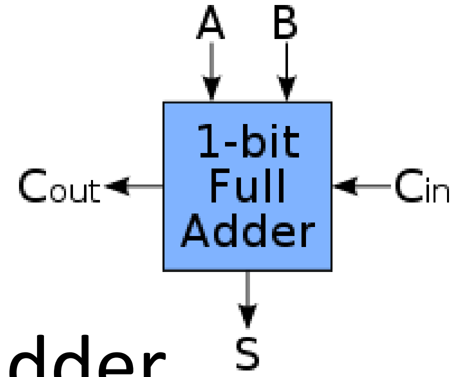
$$B_{out} = \bar{A}B + B_{in}\overline{(A \oplus B)}$$

FS vs FA



FS – Full Subtractor

FA – Full Adder



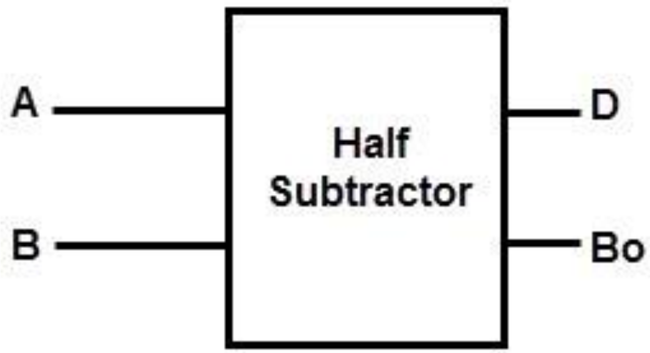
Inputs			Outputs	
A	B	B_{in}	D	B_{out}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$$D = (A \oplus B) \oplus B_{in}$$
$$B_{out} = \bar{A}B + B_{in}\overline{(A \oplus B)}$$

Inputs			Outputs	
A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = (A \oplus B) \oplus C_{in}$$
$$C_{out} = AB + C_{in}(A \oplus B)$$

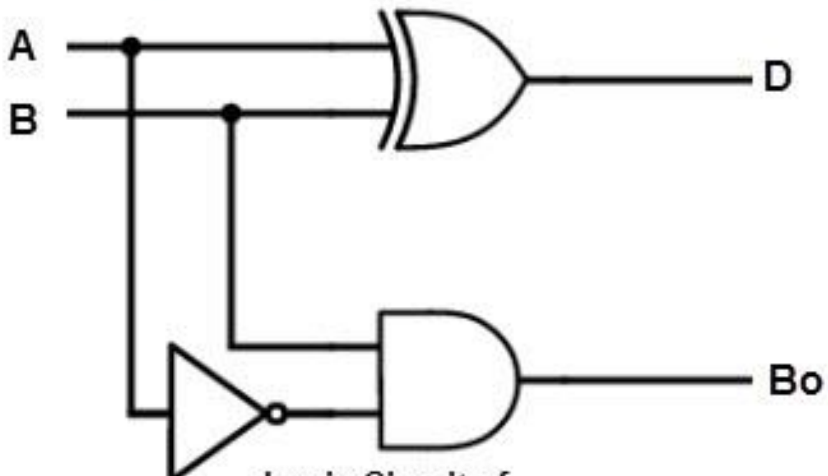
Semi scazator



Block Diagram

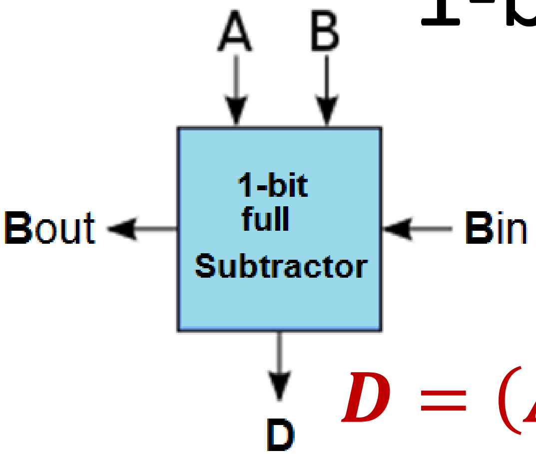
A	B	D	B_o
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Truth Table



Logic Circuit of
Half Subtractor

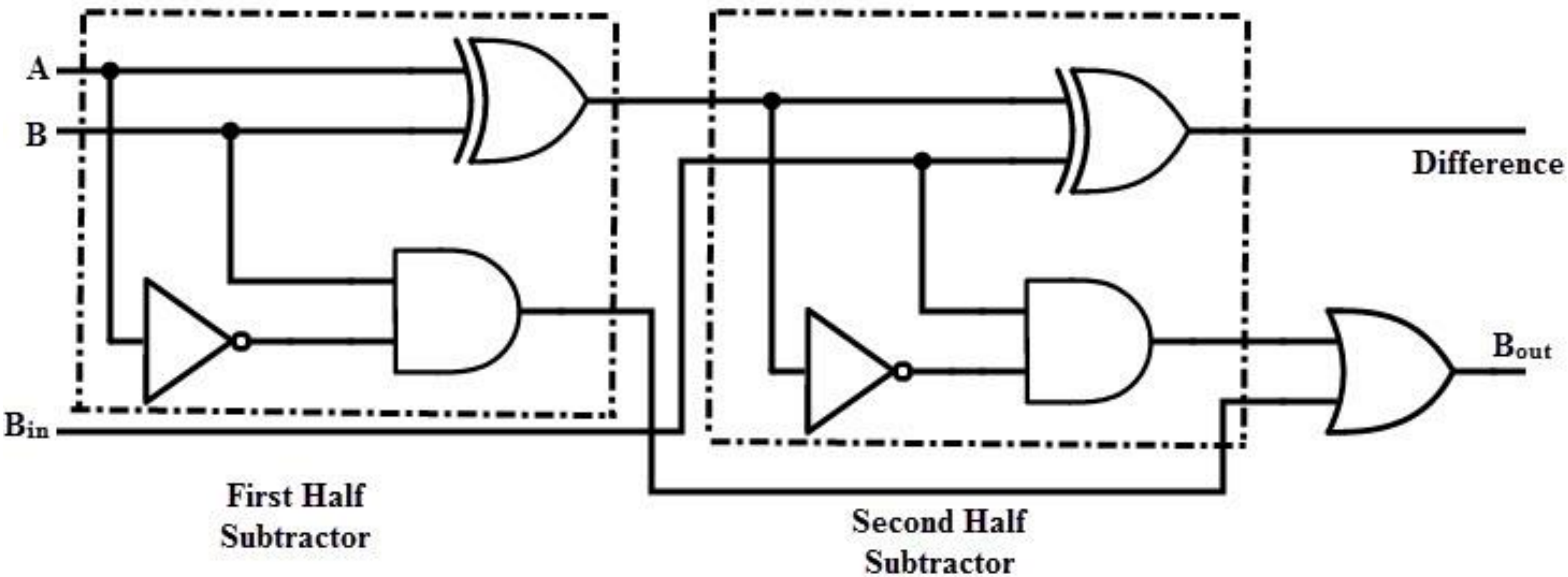
Scazatorul complet pe 1-bit: **A-B**



$$D = (A \oplus B) \oplus B_{in}$$

$$B_{out} = \bar{A}B + B_{in}\overline{(A \oplus B)}$$

Inputs			Outputs	
A	*B*	B_{in}	D	B_{out}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1



Multumesc pentru atentie!