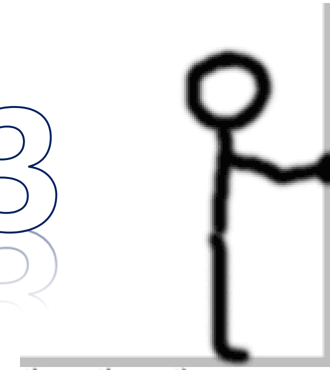
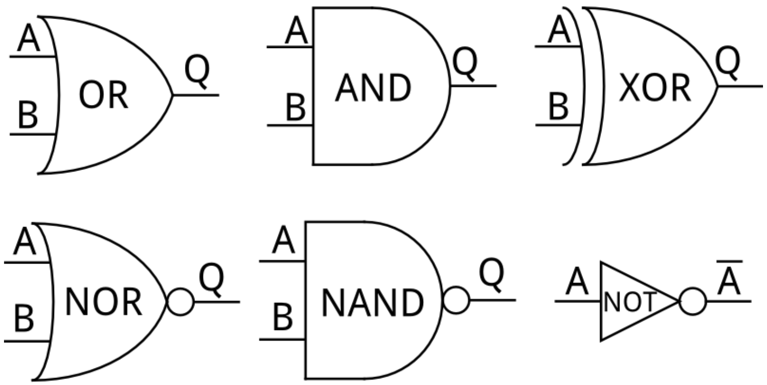
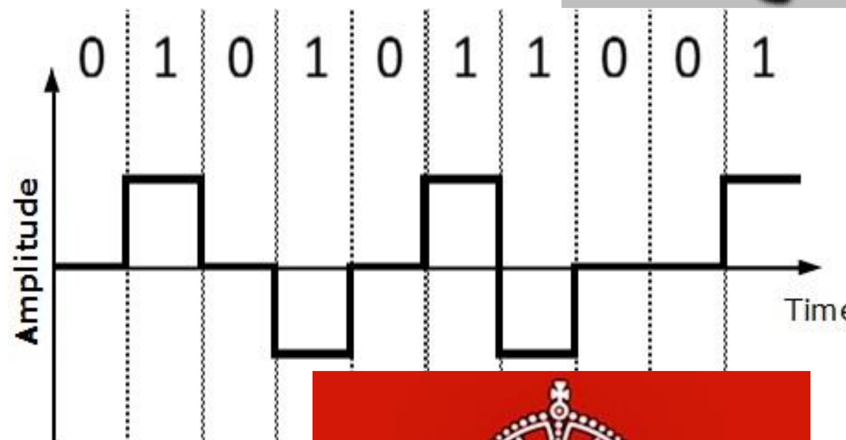
Proiectare Logica

Digital Logic Design

Curs 03







Recapitulare

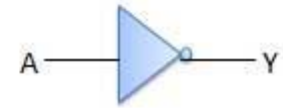
- Mr. Smith Problem (utilitatea algebrei booleene),
- Operatii logice: simboluri si notatii
- Algebra booleana: Teoreme Simple; Asociativitate, Comutativitate si Distributivitate; Reguli de de simplificare; Principiul dualitatii, Teoremele De Morgan; Termeni de consens
- Demonstrarea identitatilor folosind teoremele aglebrei booleene sau tabela de adevar

Algebra Booleana

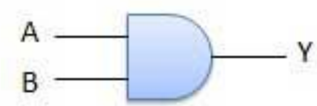
*I will **take** an **umbrella** with me if*
*it is **raining** or the weather **forecast** is **bad**.*

Raining	Bad Forecast	Umbrella
FALSE	FALSE	FALSE
FALSE	TRUE	TRUE
TRUE	FALSE	TRUE
TRUE	TRUE	TRUE

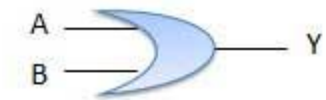
NOT → **Y=A'**

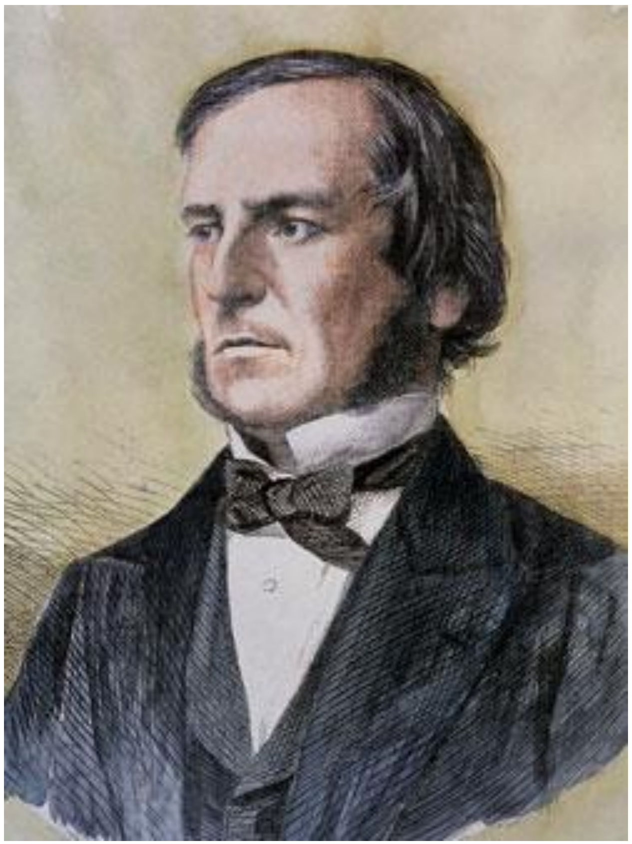


AND → **Y=A·B**



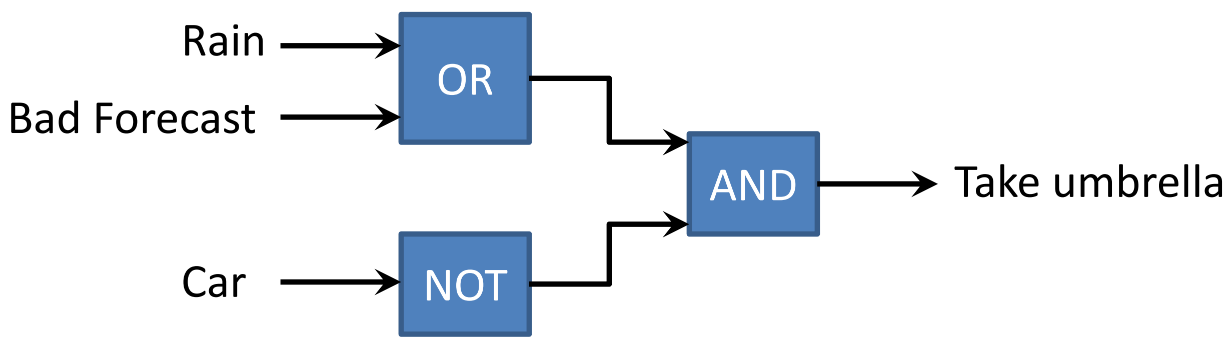
OR → **Y=A+B**



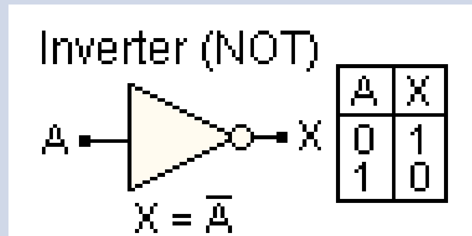
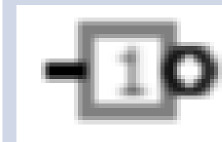
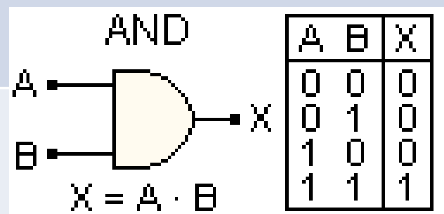
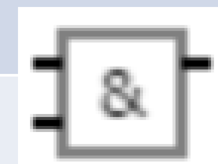
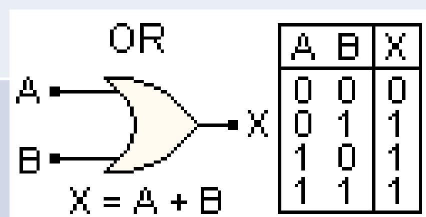
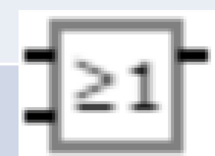
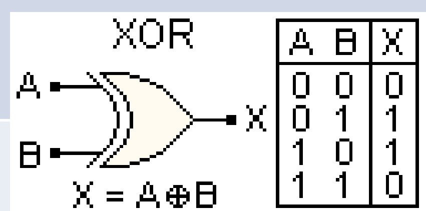
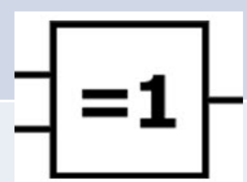
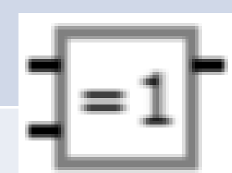


George Boole (1815-1864)

*If I do not take the **car** then I will take the **umbrella** if it is **raining** or the weather **forecast** is **bad.***



Operatii logice: simboluri si notatii

Operatie	Tip Op	Notatie	Simbol clasic	Simbol IEEE	wronex
NOT	unar	$\bar{A}$, A' sau $\neg A$			
AND	binar	$A \cdot B$			
OR	binar	$A + B$			
XOR	binar	$A \oplus B$			

Teoreme Simple:

$(\boldsymbol{A}')' = \boldsymbol{A}$: Dubla negatie == afirmatie

$\boldsymbol{A} \cdot \boldsymbol{A}' = \mathbf{0}$: O propozitie SI negatia sa ➔FALSA

$\boldsymbol{A} + \boldsymbol{A}' = \mathbf{1}$: O propozitie SAU negatia sa ➔ADEV.

Asociativitate, Comutativitate si Distributivitate

Asociativitate:

$$(A \cdot B) \cdot C = A \cdot (B \cdot C) = A \cdot B \cdot C$$

$$(A + B) + C = A + (B + C) = A + B + C$$

Comutativitate:

$$A \cdot B = B \cdot A$$

$$A + B = B + A$$

Distributivitate:

$$A \cdot (B + C) = A \cdot B + A \cdot C$$

$A + (B \cdot C) = (A + B) \cdot (A + C) \rightarrow$ Ciudata

Reguli de simplificare

- **Idempotenta:**

$$\boldsymbol{A \cdot A = A}$$

$$\boldsymbol{A + A = A}$$

- **In care apar constantele booleene:**

$\boldsymbol{A \cdot 0 \ = \ 0}$

$\boldsymbol{A \cdot 1 \ = \ A}$: 1 = elem. neutru fata de $\boldsymbol{" \cdot "}$

$\boldsymbol{A \ + \ 0 \ = \ A}$: 0 = elem. neutru fata de $\boldsymbol{" + "}$

$\boldsymbol{A \ + \ 1 \ = \ 1}$

- **Grupul cel mai larg de reguli de simplificare:**

$\boldsymbol{A + A \cdot B = A}$ si duala sa $\boldsymbol{A \cdot (A + B) = A}$

$\boldsymbol{A + A \cdot B' = A}$ si duala sa $\boldsymbol{A \cdot (A + B') = A}$

Principiul dualitatii – Dualitatea De Morgan

Daca intr-o identitate booleana interschimbam constantele

$$\mathbf{0} \Leftrightarrow \mathbf{1}$$

si operatorii

$$"\cdot" \Leftrightarrow "+"$$

atunci obtinem o noua identitate, numita identitatea duala.

Exemple:

$\boldsymbol{A} \cdot \mathbf{0} = \mathbf{0}$ si duala sa $\boldsymbol{A} + \mathbf{1} = \mathbf{1}$

Teoremele De Morgan

Un exemplu remarcabil al principiului dualitatii il reprezinta teoremele De Morgan

$$(\boldsymbol{A} \cdot \boldsymbol{B})' = \boldsymbol{A}' + \boldsymbol{B}'$$
$$(\boldsymbol{A} + \boldsymbol{B})' = \boldsymbol{A}'\boldsymbol{B}'$$

Nota: In locul operatorului **NOT** notat cu (**'**) – single quote (de ex. $\boldsymbol{A}'$) se mai foloseste simbolul $\neg$, adica $\neg\boldsymbol{A}$**.** La fel de uzitata este si notatia cu bara deasupra: $\overline{\boldsymbol{A}}$**.**

Termen de consens

- **Teorema de consens:**

Fie functia $\boldsymbol{f = A \cdot C + B \cdot C'}$. Un termen de consens este format din cele doua variabile diferite de C, adica $\boldsymbol{A \cdot B}$. Functia formata pin adaugarea acesti termen nu difera de cea initiala pentru orice valoare parametrilor: Adica $\boldsymbol{A \cdot C + B \cdot C' \equiv A \cdot C + B \cdot C' + A \cdot B}$

- **Prin definitie un termen de consens este un termen a carei prezenta nu schimba valoarea functiei.**

$$\boldsymbol{C + ABC' + AB = C + ABC'}$$

Scopul introducerii acestor termeni de consens este simplificarea. Expresia de mai sus se simplifica:

$$\boldsymbol{C + ABC' = C + AB}$$

Demonstrarea identitatilor

A+AB=A

- **Se poate face folosind teoremele**

$$\begin{aligned} A + AB &= A \cdot 1 + AB \\ &= A(B + \bar{B}) + AB \\ &= AB + A\bar{B} + AB \\ &= AB + A\bar{B} \\ &= A(B + \bar{B}) \\ &= A \end{aligned}$$

- **sau tabela de adevar**

A	B	AB	A+AB
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

Curs 03

Numere in virgula mobila

- Numerele reale sunt reprezentate in sistemele de calcul sub forma numerelor in *virgula mobila*.
- Cel mai cunoscut si raspandit standard este **IEEE -754** lansat in 1985. El a impus reprezentarea numerelor pe 32 si 64 biti (respectiv numere in simpla si dubla precizie)
- Acest standard a fost revizuit in 2008 si a devenit standard international cu numele **ISO/IEC/IEEE 60559:2011** (se poate cumpara cu ~ 700 lei).

1. IEEE - Institute of Electrical and Electronics Engineers
2. ISO - International Organization for Standardization
3. IEC - International Electrotechnical Commission

IEEE 754

- Formatul IEEE contine un set de reprezentari ale valorilor numerice si impune simboluri precum NaN, +/-Inf si altele.
- Un numar real in acest format este reprezentat prin 3 numere intregi

$$\boxed{\boldsymbol{n = (-1)^{s} \times m \times b^{e}}}$$

  - ***s*** – semnul (0 sau 1 → nr + sau -)
  - ***e*** – exponent
  - ***m*** – mantisa (significand in EN). Ea memoreaza cifrele semnificative.
- mai este precizat un numar
  - ***b*** – baza de numeratie (2 sau 10)

Formatele IEEE - 754

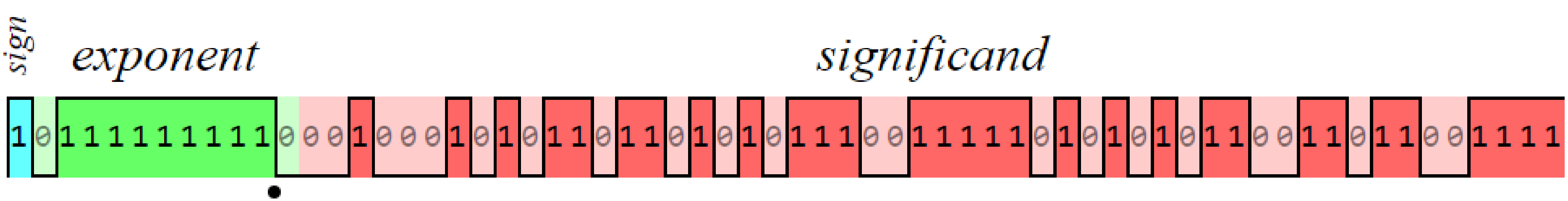
- Cele mai raspandite sunt **binari32** si **binari64**, respectiv in simpla si dubla precizie (vezi aici)

Nume	Nume usual	Baza	nr biti mantisa	Nr cifre semnif in baza 10	Nr. biti exp.	Exp. bias	E min	E max
binary16	Half precision	2	11	3.31	5	$2^4-1 = 15$	−14	+15
binary32	Single precision	2	24	7.22	8	$2^7-1 = 127$	−126	+127
binary64	Double precision	2	53	15.95	11	$2^{10}-1 = 1023$	−1022	+1023
binary128	Quadruple precision	2	113	34.02	15	$2^{14}-1 = 16383$	−16382	+16383
binary256	Octuple precision	2	237	71.34	19	$2^{18}-1 = 262143$	−262142	+262143

Bitul cel mai semnificativ al mantisei este omis in reprezentare. Ex. in **binary64** : 1s+11e+(53-1)m=64biti

Binary64 online

- Pentru reprezentarea in ***dubla precizie*** (**binary64**) exista o pagina online unde se poate vizualiza acest format (si aici pe cnic.ro).
- **binary64** foloseste **1** bit de semn, **11** biti pentru exponent si **52** biti pentru mantisa (NCM-1; adica se omite bitul cel mai semnificativ din mantisa deoarece acesta este intotdeauna 1)
- Exemplu: -0.5678_{10} se reprezinta astfel



Numar cifre semnificative in baza 10

- $NCS10$ =Numar cifre semnificative in baza 10
- NCE = Numar cifre binare exponent
- NCM = Numar cifre binare mantisa

$$\boldsymbol{NCS10 = \log_{10} 2^{NCM} \cong NCM \times log_{10} 2}$$

- Ex:
  - binary32 $NCS10 = 24 \log_{10} 2 \cong 24 \times 0.301 \cong 7.22$
  - binary64 $NCS10 = 53 \log_{10} 2 \cong 15.95$

Nume	NCM	NCE	NCS10
binary16	11	5	3.31
binary32	24	8	7.22
binary64	53	11	15.95
binary128	113	15	34.02
binary256	237	19	71.34

Numar cifre semnificative in baza 10

- $NCS10$ = Numar cifre semnificative in baza 10
- NCM = Numar cifre in baza b al mantisei

$$\boldsymbol{NCS10 = \log_{10} b^{NCM} \cong NCM \times log_{10} b}$$

Algoritm conversie in virgula mobila

- Se extrage semnul: **`if(x>=0)semn=0;else semn=1`**
- Se calculeaza deplasarea: **`offset=`**$\mathbf{2^{NCE-1}-1}$.
  De ex. in **`binary64`**: NCE=11 **`offset=`**$2^{10}-$1=**`1023`**
- Se calculeaza exponentul real:
  `er=[lg(|x|)/lg(2)]`
  adica **`er=`**partea intreaga din $\frac{\log_{10}|x|}{\log_{10}2}$
- Se determina exponentul: **`exp=er+offset`**
- Se determina partea fractionara ce intra in mantisa:
  man=$\mathbf{|x|/2^{er}-1}$
- Se trece in baza 2 aceasta parte fractionara pe $\boldsymbol{NCM-1}$ biti
- EXCEPTIE cand x=0 ➔ semn=0;exp=0;man=0

Algoritm conversie in virgula mobila

- Exemplu x=-12.0625 ➔ **semn=1**
- in **binary64**: **offset=**$2^{10}-1$=**1023**
- Se calculeaza exponentul real: **er=**partea intreaga din $\frac{\log_{10}|x|}{\log_{10} 2}$=3
- exponentul: **exp=er+offset=1026$_{10}$=10000000010$_2$**
- partea fractionara a mantisei: **man=**$\boldsymbol{|x|/2^{er}-1}$**=0.5078125$_{10}$=0.1000001…0$_2$**
- Se trece in baza 2 aceasta parte fractionara pe $\boldsymbol{NCM-1}$ =52 biti

Algoritm conversie in virgula mobila

- Exemplu x=-12.0625➔ **semn=1**
- $= -1 \times (2^3 + 2^2 + 2^{-4})$
- $= -1^1 \times 2^3 \times (2^0 + 2^{-1} + 2^{-7})$
- $= -1^1 \times 2^{1026-1023} \times (1 + 2^{-1} + 2^{-7})$
- $1\underbrace{10000000010}_{\textbf{exponent}\ 11\ \text{biti}}\underbrace{1000001000\ldots0}_{\textbf{mantisa}\ 52\ \text{biti}}$

semn 1 bit

- Alte exemple –21.375; 47.9375; 7.6875 etc ([local])

Functii booleene: forme de reprezentare

- Forma disjunctiva FD:
  - $f(A, B, C, D) = AB\bar{C} + CD + \bar{B}$

    Suma de produse

- Forma conjunctiva FC:
  - $g(A, B, C, D) = (A + B + C)(\bar{C} + \bar{D})(A + B)$

    Produs de sume

Forme canonice

$\mathbf{0 \cdot A = 0; 0 + A = A}$
dispar termenii pentru care $\boldsymbol{a_i = 0}$

- Forma canonica disjunctiva FCD:

Suma de produse

- $f(A,B,C) = a_0A'B'C' + a_1A'B'C + a_2A'BC' + \cdots + a_7ABC = \sum_{i=0}^{2^3-1} a_i m_i$, unde m_i se numeste **mintermenul** *i*.

$\mathbf{1 + A = 1; 1 \cdot A = A}$
dispar factorii pentru care $\boldsymbol{b_i = 1}$

- Forma canonica conjunctiva FCC:

Produs de sume

- *g(A, B, C)* =$(b_0 + A + B + C) \cdot (b_1 + A + B + C') + \cdots + (b_7 + A' + B' + C') = \prod_{i=0}^{2^3-1} M_i$, unde M_i se numeste **maxtermenul** *i*.

$\boldsymbol{b_0 \rightarrow A + B + C = (A'B'C')'}$
$\boldsymbol{b_1 \rightarrow A + B + C' = (A'B'C)'}$

← De Morgan

Tabele de adevar - mintermeni

- Forma analitica
  - $$f(A,B,C,D) = \overbrace{\bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C}^{\text{Mintermeni}} = m_1 + m_2 + m_3 + m_5$$
- Tabelul de adevar

Mintermen	Notatie	A	B	C	f
$\overline{A}\overline{B}\overline{C}$	m_0	0	0	0	0
$\overline{A}\overline{B}C$	m_1	0	0	1	1
$\overline{A}B\overline{C}$	m_2	0	1	0	1
$\overline{A}BC$	m_3	0	1	1	1
$A\overline{B}\overline{C}$	m_4	1	0	0	0
$A\overline{B}C$	m_5	1	0	1	1
$AB\overline{C}$	m_6	1	1	0	0
ABC	m_7	1	1	1	0

Forme canonice: exemplu

- Fie functia *f* definita de tabelul:

i	A	B	C	*f*	mintermeni (m_i)	Maxtermeni (M_i) Minfactori
0	0	0	0	**1**	**A'B'C'**	A+B+C
1	0	0	1	**0**	A'B'C	**A+B+C'**
2	0	1	0	**0**	A'BC'	**A+B'+C**
3	0	1	1	**1**	**A'BC**	A+B'+C'
4	1	0	0	**1**	**AB'C'**	A'+B+C
5	1	0	1	**0**	AB'C	**A'+B+C'**
6	1	1	0	**1**	**ABC'**	A'+B'+C
7	1	1	1	**1**	**ABC**	A'+B'+C'

FCD: $f = A'B'C' + A'BC + AB'C' + ABC' + ABC = \sum m(0,3,4,6,7)$

FCC: $f = (A + B + C')(A + B' + C)(A' + B + C') = \prod M(1,2,5)$

Diagrame Venn

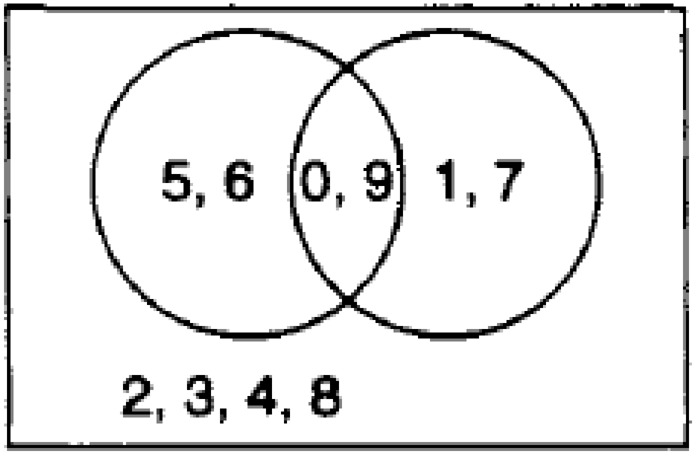


Diagrama in teoria multimilor

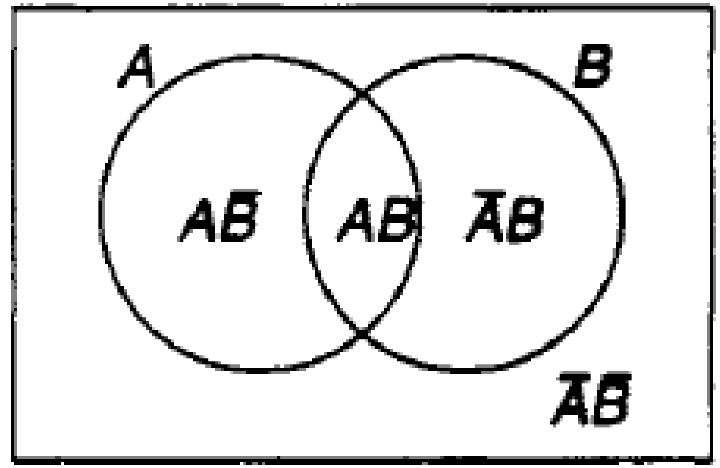
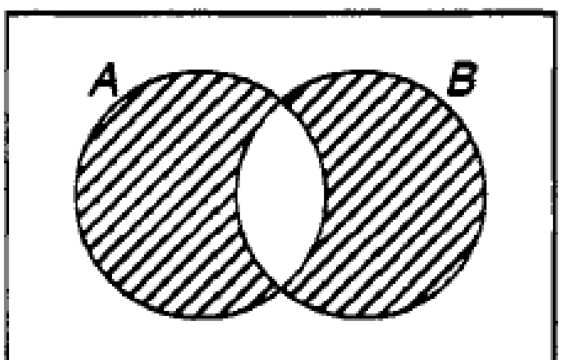
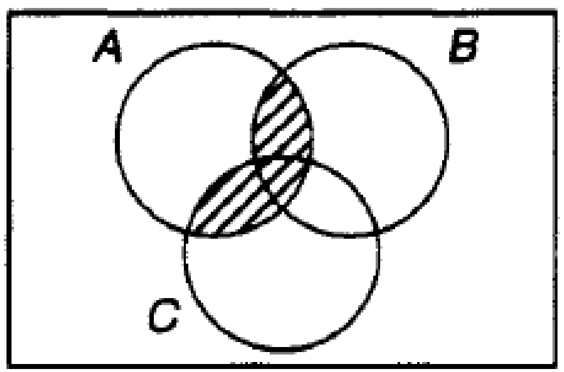


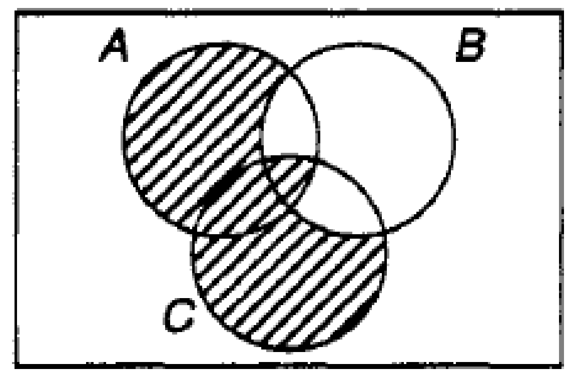
Diagrama Venn



$A\bar{B}+\bar{A}B$



$AB+AC$



$AC+\bar{B}C+A\bar{B}$

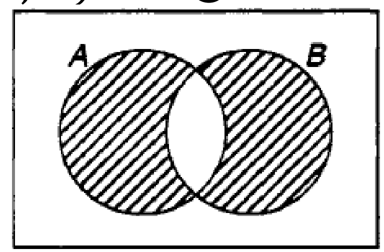
Diagrame Veitch: reprezentari canonice

- 2 variabile

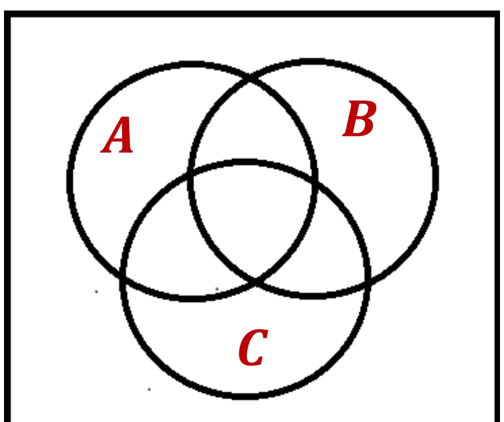
$f(A,B)$	$\overline{A}$	A
$\overline{B}$	$\overline{A}\overline{B}$	$A\overline{B}$
B	$\overline{A}B$	AB

$A \oplus B$	$A=0$	$A=1$
$B=0$	0	1
$B=1$	1	0

$$f(A,B) = A \oplus B = A\overline{B} + \overline{A}B$$



- 3 variabile



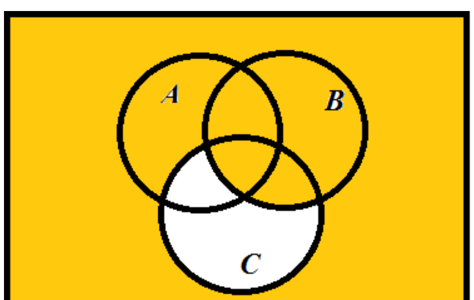
	$\overline{A}$		A		
$f(A,B,C)$	$\overline{A}\overline{B}$	$\overline{A}B$	AB	$A\overline{B}$	
$\overline{C}$	$\overline{A}\overline{B}\overline{C}$	$\overline{A}B\overline{C}$	$AB\overline{C}$	$A\overline{B}\overline{C}$	$\overline{C}$
C	$\overline{A}\overline{B}C$	$\overline{A}BC$	ABC	$A\overline{B}C$	C
	$\overline{B}$	B		$\overline{B}$	

Diagrame Veitch: reprezentari canonice

- 3 variabile

	$\overline{A}$		A		
$f(A,B,C)$	$\overline{A}\,\overline{B}$	$\overline{A}B$	AB	$A\overline{B}$	
$\overline{C}$	$\overline{A}\,\overline{B}\,\overline{C}$	$\overline{A}B\overline{C}$	$AB\overline{C}$	$A\overline{B}\,\overline{C}$	$\overline{C}$
C	$\overline{A}\,\overline{B}C$	$\overline{A}BC$	ABC	$A\overline{B}C$	C
	$\overline{B}$	B		$\overline{B}$	

$f(A,B,C)$	$\overline{A}\,\overline{B}$	$\overline{A}B$	AB	$A\overline{B}$
$\overline{C}$	1	1	1	1
C	0	1	1	0

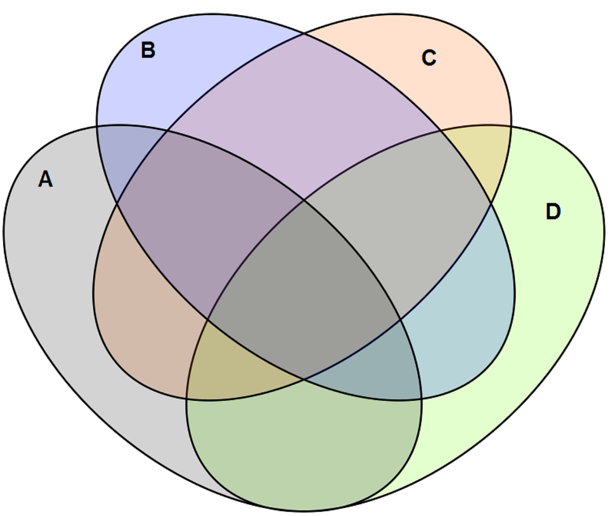


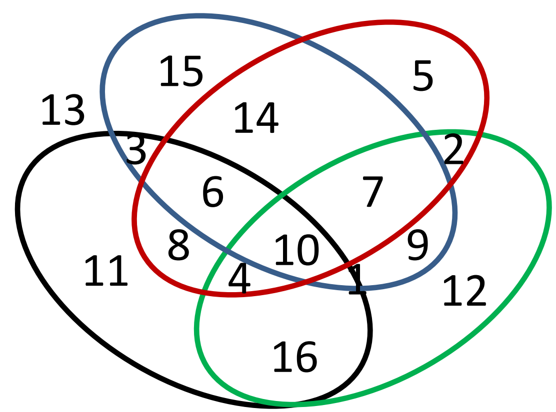
$$f(A,B,C) = B + \overline{C}$$

Diagrame Veitch: reprezentari canonice

- 4 variabile

		$\overline{A}$	$\overline{A}$	A	A	
	$f(A,B,C,D)$	$\overline{A}\overline{B}$	$\overline{A}B$	AB	$A\overline{B}$	
$\overline{D}$	$\overline{C}\overline{D}$	$\overline{A}\overline{B}\overline{C}\overline{D}$	$\overline{A}B\overline{C}\overline{D}$	$AB\overline{C}\overline{D}$	$A\overline{B}\overline{C}\overline{D}$	$\overline{C}$
D	$\overline{C}D$	$\overline{A}\overline{B}\overline{C}D$	$\overline{A}B\overline{C}D$	$AB\overline{C}D$	$A\overline{B}\overline{C}D$	$\overline{C}$
D	CD	$\overline{A}\overline{B}CD$	$\overline{A}BCD$	$ABCD$	$A\overline{B}CD$	C
$\overline{D}$	$C\overline{D}$	$\overline{A}\overline{B}C\overline{D}$	$\overline{A}BC\overline{D}$	$ABC\overline{D}$	$A\overline{B}C\overline{D}$	C
		$\overline{B}$	B	B	$\overline{B}$	

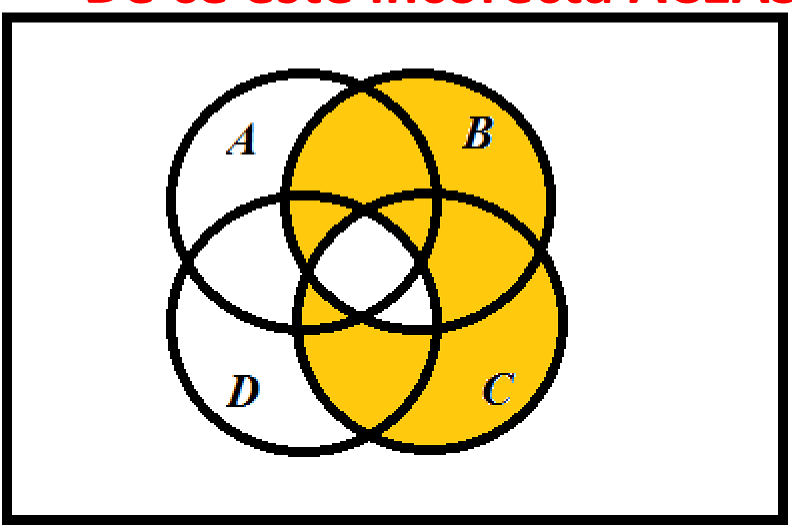


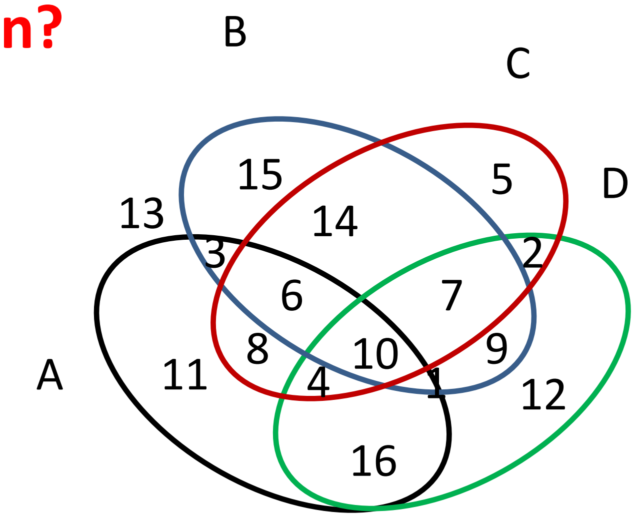


De ce este incorecta diagrama Venn?

$\bar{A}\bar{B}\bar{C}\bar{D}$ →13	$\bar{A}B\bar{C}\bar{D}$	$AB\bar{C}\bar{D}$	$A\bar{B}\bar{C}\bar{D}$ →11
$\bar{A}\bar{B}\bar{C}D$	$\bar{A}B\bar{C}D$	$AB\bar{C}D$	$A\bar{B}\bar{C}D$
$\bar{A}\bar{B}CD$	$\bar{A}BCD$	$ABCD$ →10	$A\bar{B}CD$
$\bar{A}\bar{B}C\bar{D}$	$\bar{A}BC\bar{D}$	$ABC\bar{D}$	$A\bar{B}C\bar{D}$

De ce este incorecta ACEASTA diagrama Venn?





Diagrame Venn: reprezentari canonice

- 4 variabile

		$\overline{A}$		A		
	$f(A,B,C,D)$	$\overline{A}\overline{B}$	$\overline{A}B$	AB	$A\overline{B}$	
$\overline{D}$	$\overline{C}\overline{D}$	0	1	1	0	$\overline{C}$
D	$\overline{C}D$	0	1	1	0	
	CD	1	0	0	1	C
$\overline{D}$	$C\overline{D}$	1	1	1	1	
		$\overline{B}$	B		$\overline{B}$	

$$f(A,B,C,D) = B\overline{C} + \overline{B}C + C\overline{D}$$

Tema de antrenament

- Problemele 2.1-2.14 de la pagina 41 din cursul de referinta : B. Holdsworth and R.C. Woods, *Digital Logic Design*, 4th Edition, 1999