Presentation of problem T1 (9 points): The Maribo Meteorite

Definitions

Meteoroid. A small particle (typically smaller than 1 m) from a comet or an asteroid.

Meteorite: A meteoroid that impacts the ground

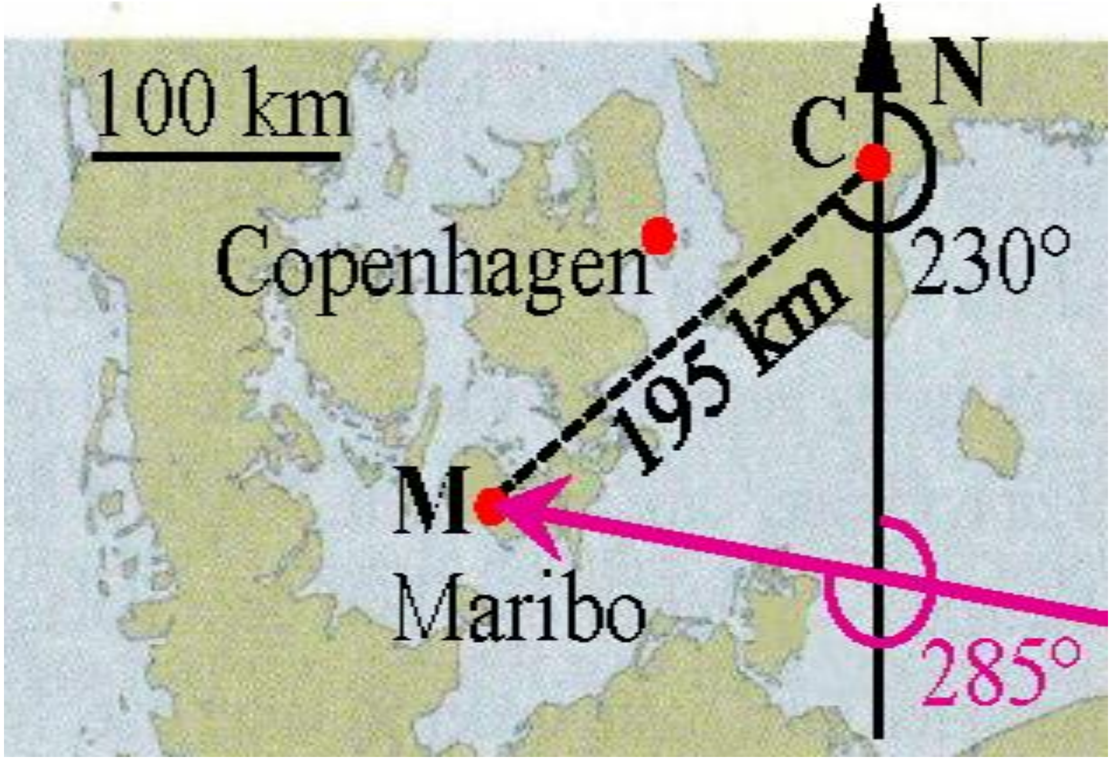
Facts about the Maribo Meteorite

- 0.025 kg meteorite found six weeks after impact near the town Maribo in southern Denmark
- Very high speed through atmosphere
- Old, formed shortly after the birth of the solar system
- Possibly a part of Comet Encke

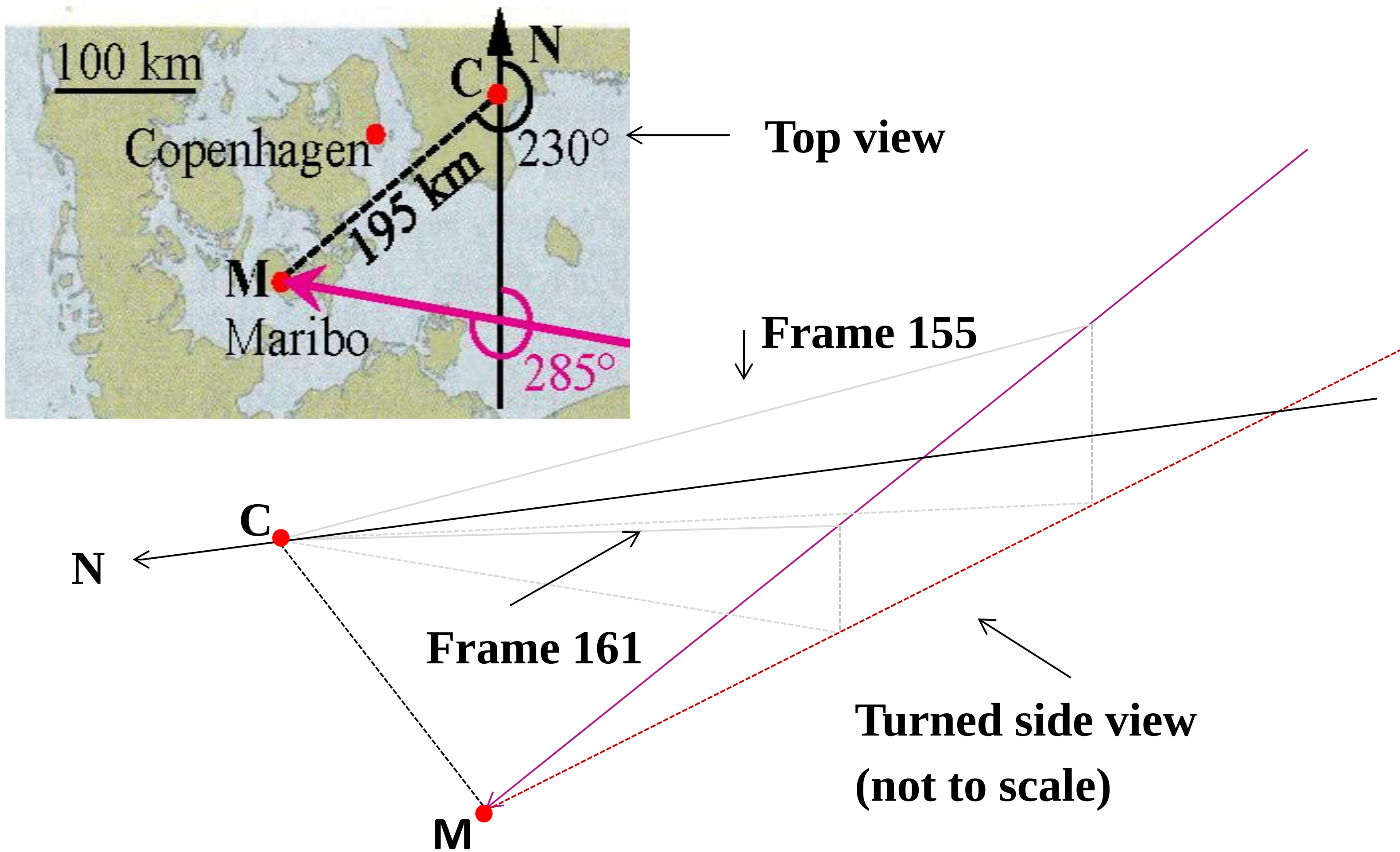
17 January 2009, Southern Sweden
Observation of meteorite

17 January 2009, Southern Sweden
Observation of a meteoroid

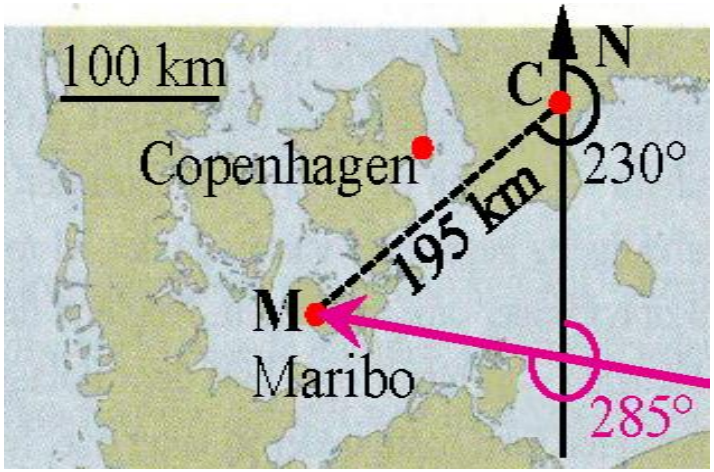
The Speed of Maribo



The Speed of Maribo



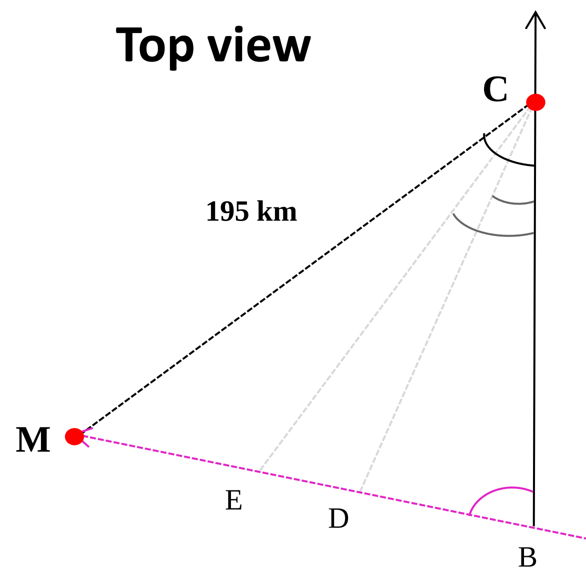
The Speed of Maribo



(b)

Frame	Time	Azimuth	Altitude
155	1.46 s	215°	19.2°
161	2.28 s	221°	14.7°
Landing at M		230°	0.0°

Top view



Use the data to calculate the speed of Maribo

Trigonometry in triangles

28.6 km/s

Through the atmosphere and melting?

Friction force from air on a supersonic meteoroid *is complicated* and depends on

- Shock wave
- Shape
- Velocity
- Temperature of atmosphere (varying)
- Density of atmosphere (varying)
- Possible rotation of meteoroid
- … and more

Simple model
(used for both sub- and supersonic motion in the literature):

$$F = k\,\rho_{atm} A\, v^2$$

(use ρ_{atm} constant)

Through the atmosphere and melting?

Mass:	30 kg
Radius:	0.13 m
Temperature inside:	200 K
Speed:	2.91×10^4 m/s
Density of atmosphere (constant):	$4.1 \times 10^{-3} \frac{\text{kg}}{\text{m}^3}$
Friction coefficient:	0.60
Melting point, stony meteorite:	1700 K
Specific melting heat:	2.56×10^5 J/kg

Estimate time for a speed reduction of 10 % →

$$m_M \frac{dv}{dt} = -k\rho_{atm}\pi R_M^2 v^2$$

Integrate and solve for t: $t = 0.9\ s$

How many times larger is the initial kinetic energy of the meteorite as compared to the energy needed to melt it completely ?

$$\frac{E_{kin}}{E_{melt}} = \frac{\frac{1}{2} v_M^2}{c_{sm}(T_{sm} - T_0) + L_{sm}} = 2.1 \times 10^2 \gg 1$$

Heating of Maribo during its fall

- Enters the atmosphere with supersonic speed
- Appears as a fireball because of surrounding air is glowing

How thick a layer of Maribo was heated significantly?

Physics: Heat conduction through sphere, but too complicated for the students, so instead we ask for dimensional (unit) analysis:

Falling time t
Density of Maribo ρ_{sm}
Thermal conductivity k_{sm}
Specific heat capacity c_{sm}
Thickness of heated shell x

$$x = t^{\alpha}\rho_{sm}^{\beta}c_{sm}^{\gamma}k_{sm}^{\delta}$$

$$x = \sqrt{k_{sm}t/\rho_{sm}c_{sm}}$$

Calculate x after t = 5 s, and determine x/R_{sm}

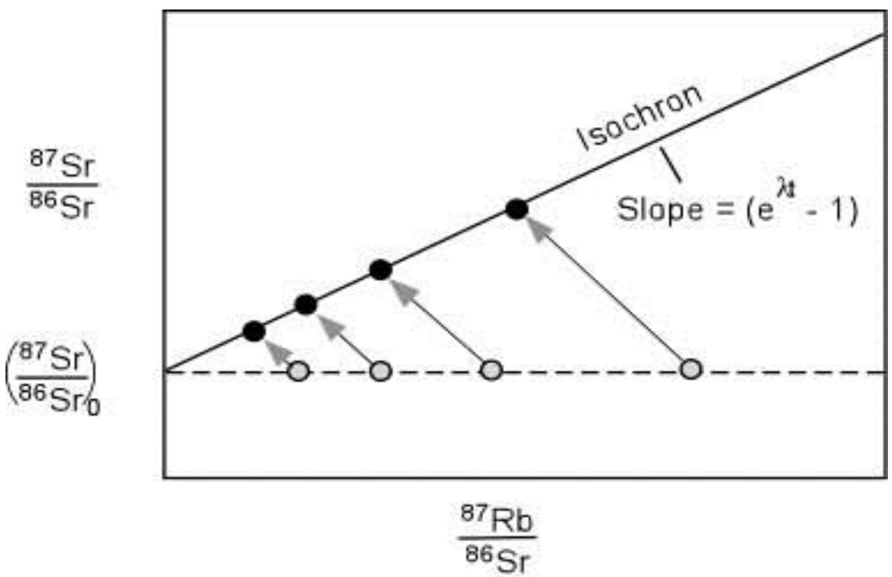
$$x = 1.6 \text{ mm and } \frac{x}{R_M} = 0.012$$

The age of a meteorite

Use radiometric dating to determine the age of a meteorite:
Many possibilities, but here we study the decay ${}^{87}_{37}\mathrm{Rb} \rightarrow {}^{87}_{38}\mathrm{Sr}$

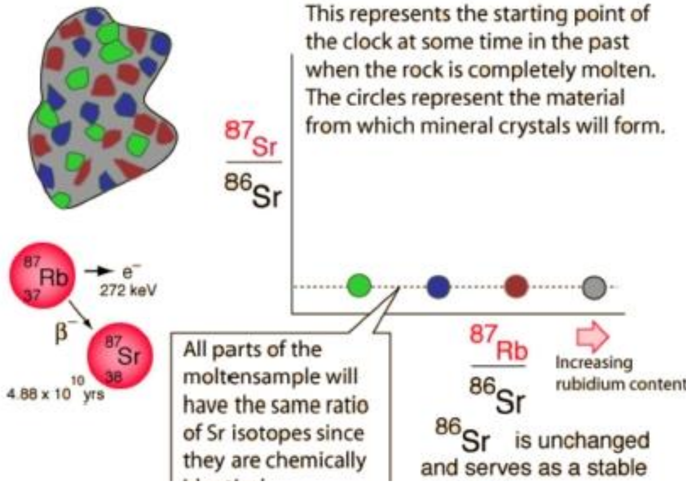
Write down the decay scheme → ${}^{87}_{37}\mathrm{Rb} \rightarrow {}^{87}_{38}\mathrm{Sr} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}_\mathrm{e}$

The ratio ${}^{87}\mathrm{Sr}/{}^{86}\mathrm{Sr}$ was identical for alle minerals in meteorite when it was formed, but the ratio ${}^{87}\mathrm{Rb}/{}^{86}\mathrm{Sr}$ was different for different minerals in the meteorite, thus:



The age of Maribo

1.



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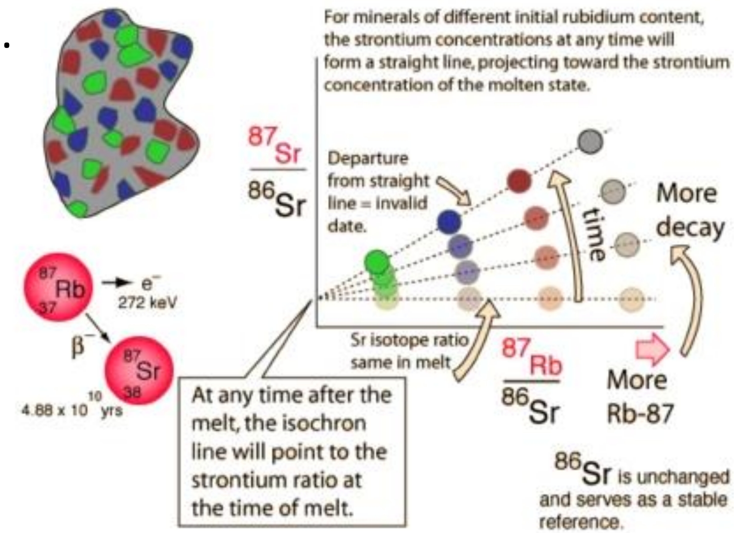

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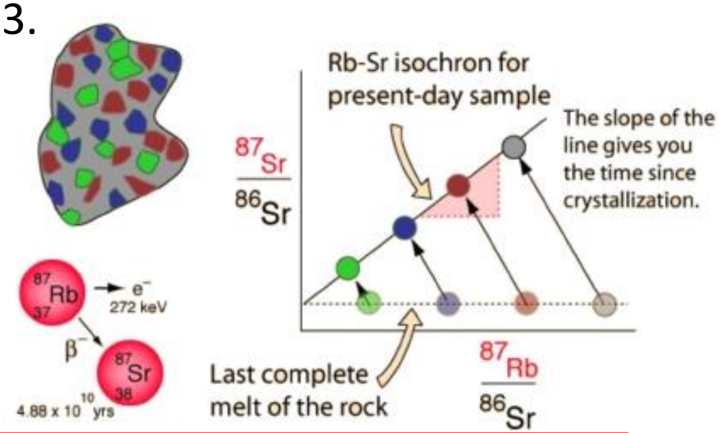


Derive the equation for the isochron straight line

$N_{87\mathrm{Rb}}(t) = N_{87\mathrm{Rb}}(0)\mathrm{e}^{-\lambda t}$ and

Rb→Sr: $N_{87\mathrm{Sr}}(t) = N_{87\mathrm{Sr}}(0) + [N_{87\mathrm{Rb}}(0) - N_{87\mathrm{Rb}}(t)]$.

Thus $N_{87\mathrm{Sr}}(t) = N_{87\mathrm{Sr}}(0) + (\mathrm{e}^{\lambda t} - 1)N_{87\mathrm{Rb}}(t)$,

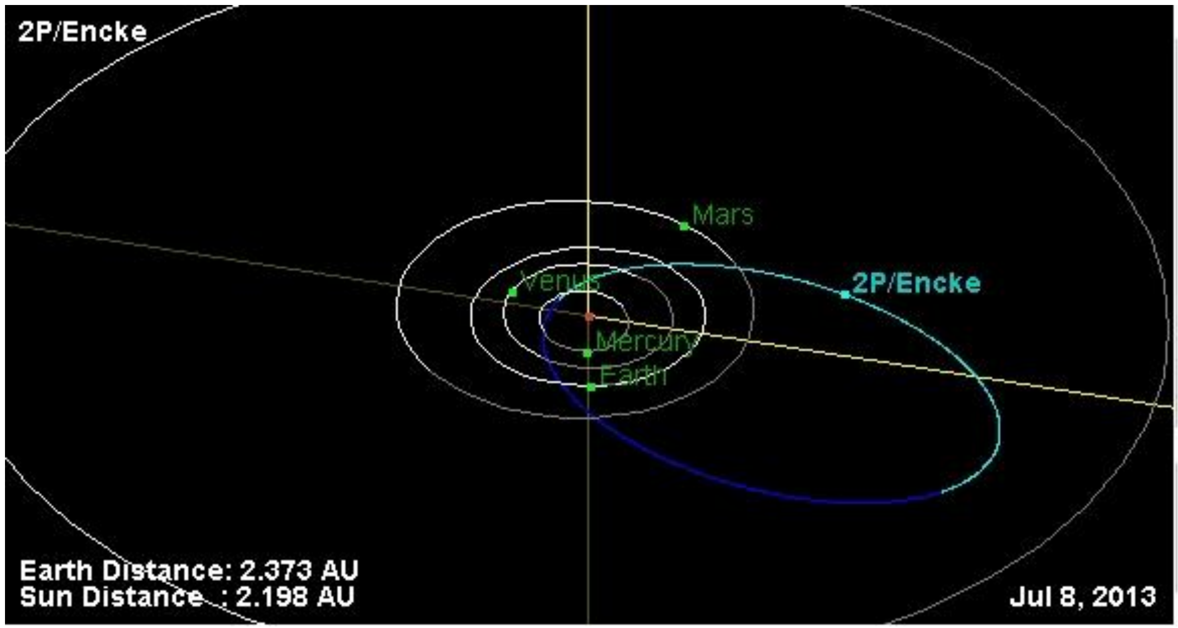
and dividing by $N_{86\mathrm{Sr}}$ we obtain

$$\frac{N_{87\mathrm{Sr}}(t)}{N_{86\mathrm{Sr}}} = \frac{N_{87\mathrm{Sr}}(0)}{N_{86\mathrm{Sr}}} + (\mathrm{e}^{\lambda t} - 1)\frac{N_{87\mathrm{Rb}}(t)}{N_{86\mathrm{Sr}}}.$$

Determine the age of a meteorite from a given measured isochron line

3.4×10^9 year

Comet Encke
from which Maribo may originate



Given minimum and maximum distance between comet Encke and the Sun

$a = \frac{1}{2}(a_{\min} + a_{\max})$ and Keplers 3. law

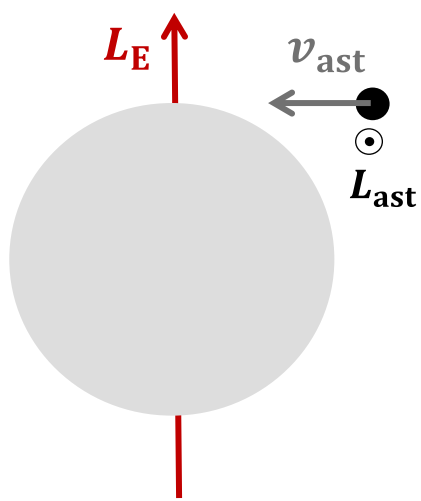
Calculate the orbital period of comet Encke → 3.3 years

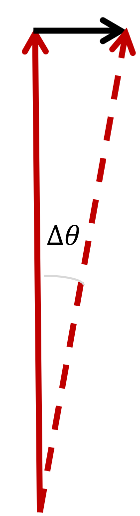
Consequences of an asteroid impact on Earth

- Assume asteroid (Chicxulub Crater type) hits the Earth.
- Given density ($3.0 \times 10^3 \mathrm{kg/m^3}$), radius (5 km) and final speed (2.5×10^4 m/s) of the asteroid.
- Assume totally inelastic collision with Earth, modeled as a sphere with moment of inertia 0.83 times that of a homogeneous sphere.

Hit on the North Pole

Find the maximum change in angular orientation of the axis of the Earth after the impact.





Use conservation of total angular momentum

$$\Delta\theta = 4.27 \times 10^{-8} \text{ rad}$$

$$R_E \Delta\theta = 0.27 \text{ m}$$

Consequences of an asteroid impact on Earth

- Assume asteroid (Chicxulub Crater type) hits the Earth.
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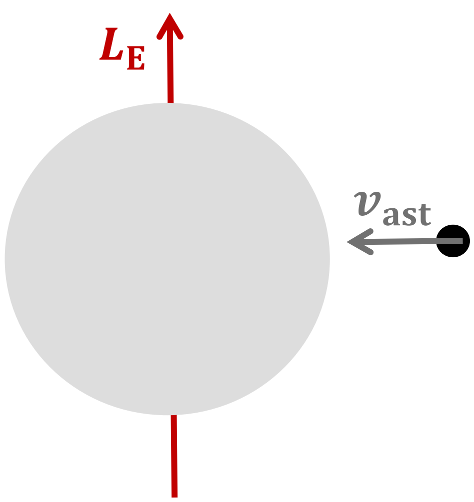
- Assume totally inelastic collision with Earth, modeled as a sphere with moment of inertia 0.83 times that of a homogeneous sphere.

Hit on the Equator in a vertical impact

Find the maximum change in the duration of one revolution of the Earth after the impact.



Use conservation of total angular momentum

$$\Delta(I\omega) = 0$$

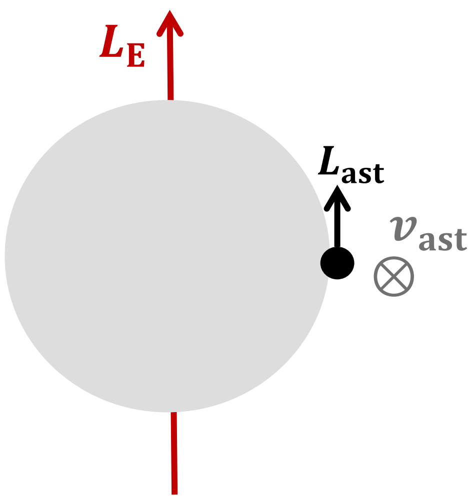
$$\Delta\tau_E = 6.84 \times 10^{-5}\ \mathrm{s}$$

Consequences of an asteroid impact on Earth

- Assume asteroid (Chicxulub Crater type) hits the Earth.
- Given density ($3.0 \times 10^3 \mathrm{kg/m^3}$), radius (5 km) and final speed (2.5×10^4 m/s) of the asteroid.
- Assume totally inelastic collision with Earth, modeled as a sphere with moment of inertia 0.83 times that of a homogeneous sphere.

Hit on the Equator in a tangential impact

Find the maximum change in the duration of one revolution of the Earth after the impact.



Use conservation of total angular momentum

$$L_E + L_{ast} = (I_E + \Delta I_E)(\omega_E + \Delta\omega_E)$$

$$\Delta\tau_E = -3.62 \times 10^{-3} \mathrm{s}$$

Minimum impact speed

Look at a celestial body, gravitationally bound in the solar system, which impacts the surface of the Earth with speed v_{imp}. Disregard friction.

Calculate the smallest possible value of v_{imp}

Escape velocity from Earth, $v_{\mathrm{min,imp}} = \sqrt{2GM_{\mathrm{E}}/R_{\mathrm{E}}} = 11.2 \times 10^3\ \mathrm{m/s}$

Maximum impact speed

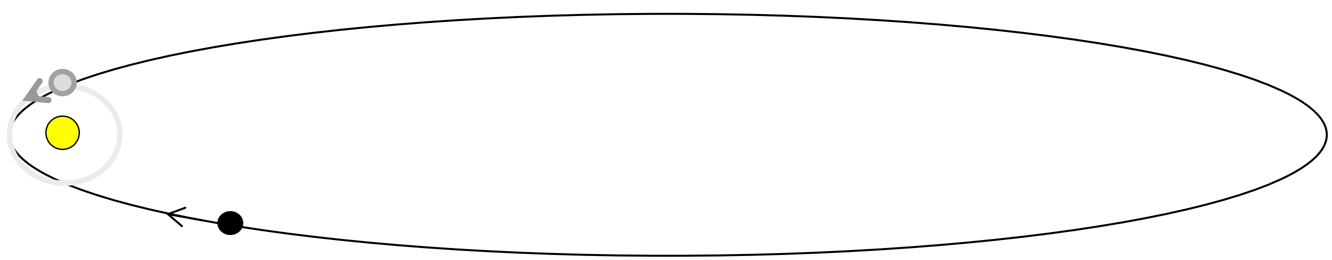
Look at a celestial body, gravitationally bound in the solar system, which impacts the surface of the Earth with speed v_{imp} . Disregard friction.

Calculate the largest possible value of v_{imp}

Very elongated orbit gives maximum velocity of body at perihelion; near-escape from surface of Earth and Sun from distance a_{E}.

Three contributions:

1) Velocity of body at distance a_{E}, $v_{\mathrm{i}} = \sqrt{\frac{2Gm_S}{a_E}} = 42.1\ \mathrm{km/s}$
2) Orbital velocity of Earth, $v_{\mathrm{E}} = \frac{2\pi a_{\mathrm{E}}}{1\ \mathrm{year}} = 29.8\ \mathrm{km/s}$
3) Gravitational attraction of Earth, $\frac{1}{2}(v_{\mathrm{i}} + v_{\mathrm{E}})^2 = -\frac{Gm_E}{R_E} + \frac{1}{2}\left(v_{\mathrm{imp}}^{\mathrm{max}}\right)^2$



$$v_{\mathrm{max,imp}} = \sqrt{(v_{\mathrm{b}} + v_{\mathrm{E}})^2 + 2Gm_{\mathrm{E}}/R_{\mathrm{E}}} = 72.8 \times 10^3\ \mathrm{m/s}$$

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The IPhO syllabus in relation to the problem

Problem	Syllabus
1.1 Speed of Maribo	1. a) Foundation of kinematics of a point mass
1.2a Speed reduction	1. b) Newton's laws
1.2b Melting?	4. a) Heat
1.3 Heat transfer	General d) (SI units)
1.4 Radioactive dating	11. c) Alpha-, beta- and gamma-decays, halflife and exponential decay,
1.5 Comet Encke	1. f) Kepler's laws
1.6 Asteroid impact	2. b) Motion of rigid bodies, translation, rotation, angular velocity, angular acceleration, conservation of angular momentum
1.7 Minimum and maximum impact speed	1. d) Conservation of energy, , conservation of linear momentum 1. e) The law of gravitation, potential energy and work in a gravitational field 1. f) Centripetal acceleration, Kepler's laws