Wednesday, April 27th, 2005

Please read this first:

1. The time available is 5 hours for the theoretical competition.
2. Use only the pen provided.
3. Use only the front side of the paper.
4. Begin each part of the problem on a separate sheet.
5. For each question, in addition to the ***blank sheets*** where you may write, there is an **a*nswer form*** where you *must* summarize the results you have obtained. Numerical results should be written with as many digits as are appropriate to the given data.
6. Write on the blank sheets of paper whatever you consider is required for the solution of the question. Please use *as little text as possible*; express yourself primarily in equations, numbers, figures, and plots.
7. Fill in the boxes at the top of each sheet of paper used by writing your ***Country No*** and ***Country Code,*** your student number (***Student No***), the number of the question ***(Question No)***, the progressive number of each sheet ***(Page No),*** and the total number of blank sheets used for each question ***(Total No of pages).*** Write the question number and the section letter of the part you are answering at the top of each sheet. If you use some blank sheets of paper for notes that you do not wish to be marked, put a large X across the entire sheet and do not include it in your numbering.
8. At the end of the exam, arrange all sheets for each problem *in the following order;*
   - answer form
   - used sheets in order
   - the sheets you do not wish to be marked
   - unused sheets and the printed question

   Place the papers inside the envelope and leave everything on your desk. You are not allowed to take *any* sheets of paper out of the room.

Question 1

1A. SPRING CYLINDER WITH MASSIVE PISTON

Consider $n=2$ moles of ideal Helium gas at a pressure P_0, volume V_0 and temperature T_0 = 300 K placed in a vertical cylindrical container (see Figure 1.1). A moveable frictionless horizontal piston of mass m = 10 kg (assume g = 9.8 m/s^2) and cross section A = 500 cm^2 compresses the gas leaving the upper section of the container void. There is a vertical spring attached to the piston and the upper wall of the container. Disregard any gas leakage through their surface contact, and neglect the specific thermal capacities of the container, piston and spring. Initially the system is in equilibrium and the spring is unstretched. Neglect the spring's mass.

a. Calculate the frequency f of small oscillation of the piston, when it is slightly displaced from equilibrium position. (**2 points**)

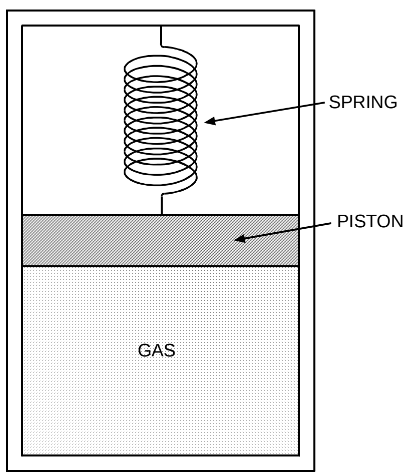

Figure 1.1

b. Then the piston is pushed down until the gas volume is halved, and released with zero velocity. calculate the value(s) of the gas volume when the piston speed is $\sqrt{\dfrac{4gV_0}{5A}}$ (**3 points**)

Let the spring constant $k = mgA/V_0$. All the processes in gas are adiabatic. Gas constant R = 8.314 JK^{-1}mol^{-1}. For mono-atomic gas (Helium) use Laplace constant $\gamma = 5/3$.

1B. THE PARAMETRIC SWING (5 points)

A child builds up the motion of a swing by standing and squatting. The trajectory followed by the center of mass of the child is illustrated in Fig. 1.2. Let r_u be the radial distance from the swing pivot to the child's center of mass when the child is standing, while r_d is the radial distance from the swing pivot to the child's center of mass when the child is squatting. Let the ratio of r_d to r_u be $2^{1/10} = 1.072$, that is the child moves its center of mass by roughly 7% compared to its average radial distance from the swing pivot.

To keep the analysis simple it is assumed that the swing be mass-less, the swing amplitude is sufficiently small and that the mass of the child resides at its center of mass. It is also assumed that the transitions from squatting to standing (the A to B and the E to F transitions) are fast compared to the swing cycle and can be taken to be instantaneous. It is similarly assumed that the squatting transitions (the C to D and the G to H transitions) can also be regarded as occurring instantaneously.

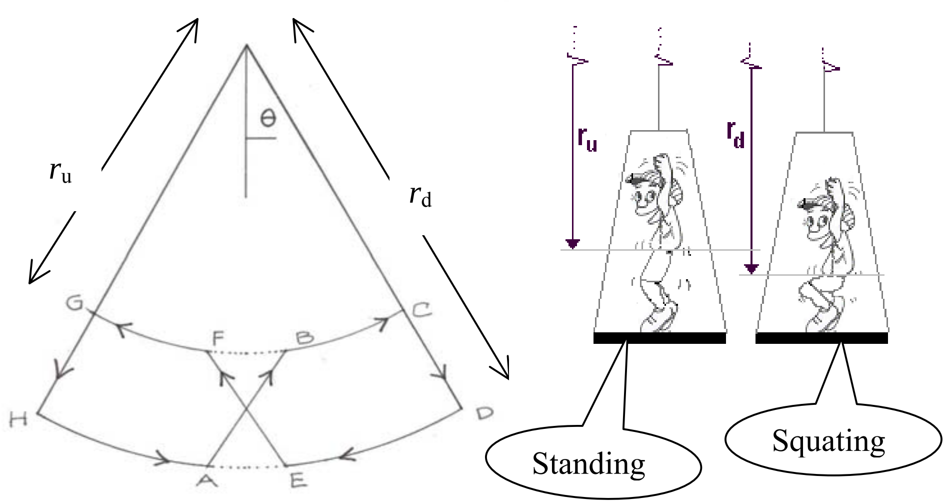

Figure 1.2

How many cycles of this maneuver does it take for the child to build up the amplitude (or the maximum angular velocity) of the swing by a factor of two?

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ANSWER FORM

1A

a)

f (formula) =

f = Hz

b)

V_{gas} *(formula)* =
Value(s) of gas volume =

1B

N (number of cycles) =

Question 2 MAGNETIC FOCUSING

There exist many devices that utilize fine beams of charged particles. The cathode ray tube used in oscilloscopes, in television receivers or in electron microscopes. In these devices the particle beam is focused and deflected in much the same manner as a light beam is in an optical instrument.

Beams of particles can be focused by electric fields or by magnetic fields. In problem 2A and 2B we are going to see how the beam can be focused by a magnetic field.

2A. MAGNETIC FOCUSING SOLENOID (4 points)

Figure 2.1 shows an electron gun situated inside (near the middle) a long solenoid. The electrons emerging from the hole on the anode have a small transverse velocity component. The electron will follow a helical path. After one complete turn, the electron will return to the axis connecting the hole and point F. By adjusting the magnetic field B inside the solenoid correctly, all the electrons will converge at the same point F after one complete turn. Use the following data:

- The voltage difference that accelerates the electrons $V = 10$ kV
- The distance between the anode and the focus point F, $L = 0.50$ m
- The mass of an electron $m = 9.11 \times 10^{-31}$ kg
- The charge of an electron $e = 1.60 \times 10^{-19}$ C
- $\mu_0 = 4\pi \times 10^{-7}$ H/m
- Treat the problem non-relativistically

a) Calculate B so that the electron returns to the axis at point F after one complete turn. (**3 points**)

b) Find the current in the solenoid if the latter has 500 turns per meter. (**1 point**)

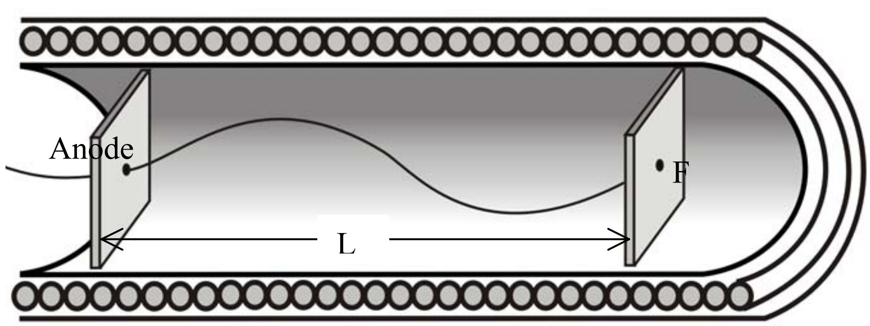

Figure 2.1

2B. MAGNETIC FOCUSING (FRINGING FIELD) (6 points)

Two pole magnets positioned on horizontal planes are separated by a certain distance such that the magnetic field between them be B in vertical direction (see Figure 2.2). The poles faces are rectangular with length l and width w. Consider the fringe field near the edges of the poles (fringe field is field particularly associated to the edge effects). Suppose the extent of the fringe field is b (see Fig. 2.3). The fringe field has two components B_x **i** and B_z **k**. For simplicity assume that $|B_x| = B|z|/b$ where z=0 is the mid plane of the gap, explicitly:

- when the particle enters the fringe field $B_x = +B\,z/b$,
- when the particle enters the fringe field after traveling through the magnet, $B_x = -Bz/b$

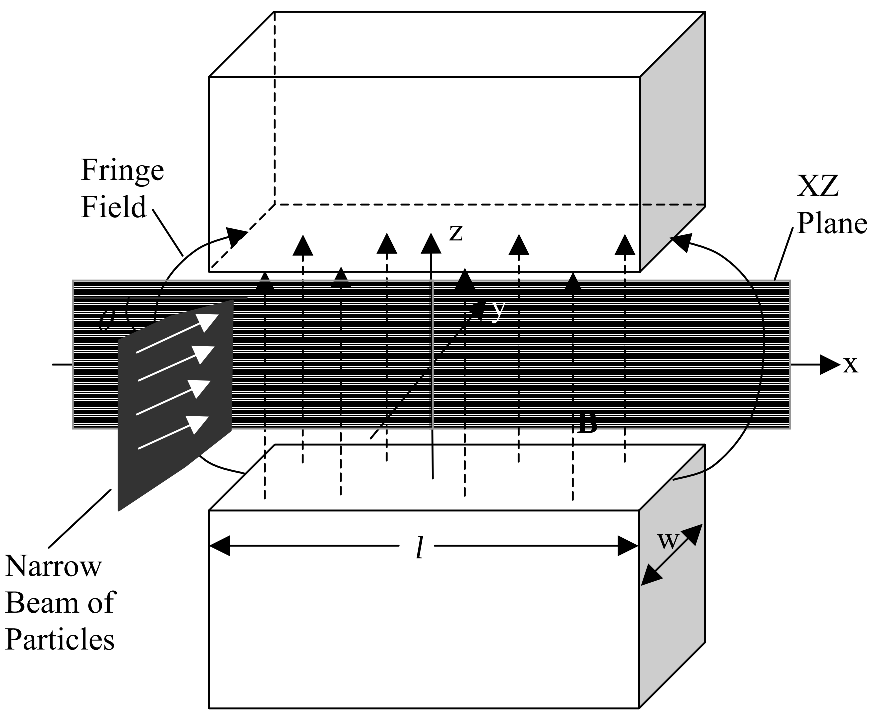

Fig.2.2: Overall view (note that θ is very small).

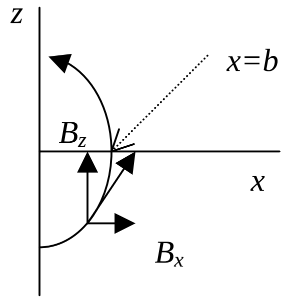

Figure 2.3. Fringe field

A parallel narrow beam of particles, each of mass m and positive charge q enters the magnet (near the center) with a high velocity v parallel to the horizontal plane. The vertical size of the beam is comparable to the distance between the magnet poles. A certain beam enters the magnet at an angle θ from the center line of the magnet and leaves the magnet at an angle $-\theta$ (see Figure 2.4. Assume θ is very small). Assume that the angle θ with which the particle enters the fringe field is the same as the angle θ when it enters the uniform field.

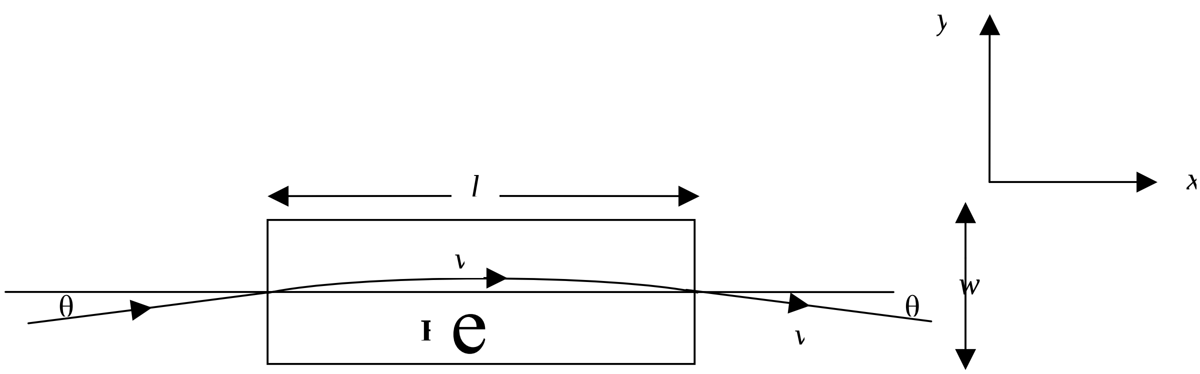

Figure 2.4. Top view

The beam will be focused due to the fringe field. Calculate the approximate focal length if we define the focal length as illustrated in Figure 2.5 (assume $b<<l$ and assume that the z-component of the deflection in the uniform magnetic field B is very small).

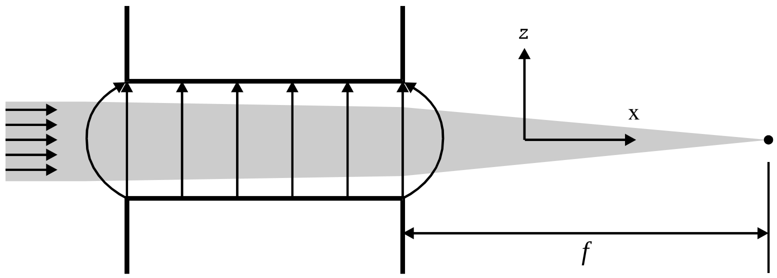

Figure 2.5. Side view

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ANSWER FORM

2A

a)

B (formula) =

B = mT

b)

I (formula) =

I = ampere

2B

Focus length (formula) =

Question 3 LIGHT DEFLECTION BY A MOVING MIRROR

Reflection of light by a relativistically moving mirror is not theoretically new. Einstein discussed the possibility or worked out the process using the Lorentz transformation to get the reflection formula due to a mirror moving with a velocity $\vec{v}$. This formula, however, could also be derived by using a relatively simpler method. Consider the reflection process as shown in Fig. 3.1, where a plane mirror M moves with a velocity $\vec{v} = v\hat{e}_x$ (where $\hat{e}_x$ is a unit vector in the x-direction) observed from the lab frame F. The mirror forms an angle ϕ with respect to the velocity (note that $\phi \leq 90^0$, see figure 3.1). The plane of the mirror has **n** as its normal. The light beam has an incident angle α and reflection angle β which are the angles between $\vec{n}$ and the incident beam 1 and reflection beam $1'$, respectively in the laboratory frame F. It can be shown that,

$$\sin\alpha - \sin\beta = \frac{v}{c}\sin\phi\sin(\alpha+\beta) \qquad (1)$$

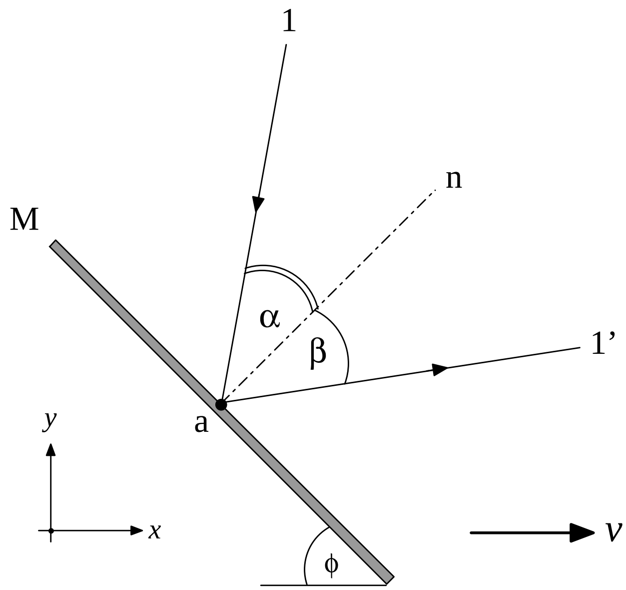

Figure 3.1. Reflection of light by a relativistically moving mirror

3A. Einstein's Mirror (2.5 points)

About a century ago Einstein derived the law of reflection of an electromagnetic wave by a mirror moving with a constant velocity $\vec{v} = -v\,\hat{e}_x$ (see Fig. 3.2). By applying the Lorentz transformation to the result obtained in the rest frame of the mirror, Einstein found that:

$$\cos\beta = \frac{\left(1+\left(\frac{v}{c}\right)^2\right)\cos\alpha - 2\frac{v}{c}}{1-2\frac{v}{c}\cos\alpha+\left(\frac{v}{c}\right)^2} \qquad (2)$$

Derive this formula using Equation (1) without Lorentz transformation!

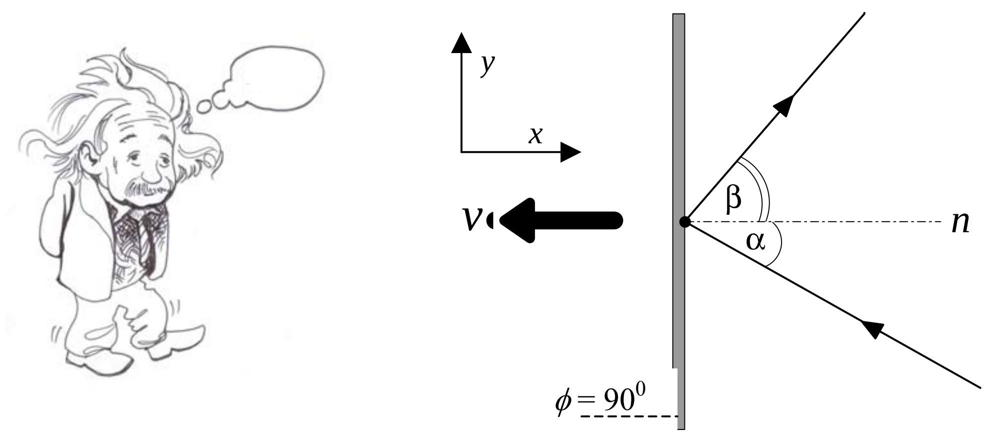

Figure 3.2. Einstein mirror moving to the left with a velocity *v*.

3B. Frequency Shift (2 points)

In the same situation as in 3A, if the incident light is a monochromatic beam hitting M with a frequency f, find the new frequency f' after it is reflected from the surface of the moving mirror. If $\alpha = 30^0$ and $v = 0.6\,c$ in figure 3.2, find frequency shift Δf in percentage of f.

3C. Moving Mirror Equation (5.5 Points)

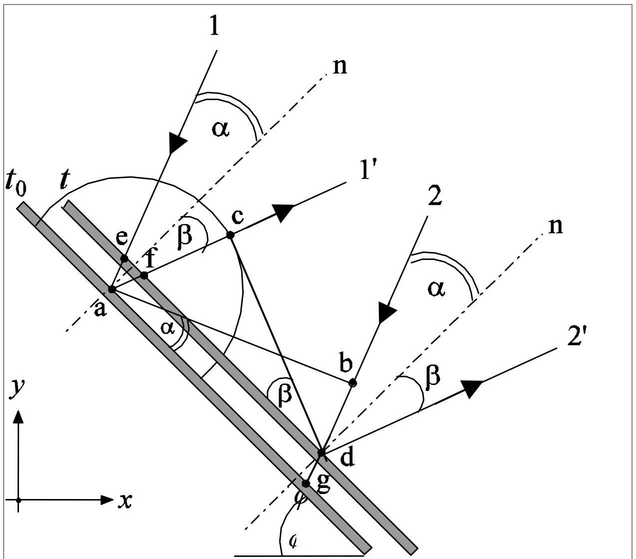

Figure 3.3.

Figure 3.3 shows the positions of the mirror at time t_0 and t. Since the observer is moving to the left, the mirror moves relatively to the right. Light beam 1 falls on point *a* at t_0 and is reflected as beam $1'$. Light beam 2 falls on point *d* at t and is reflected as beam $2'$. Therefore, $\overline{ab}$ is the wave front of the incoming light at time t_0. The atoms at point are disturbed by the incident wave front $\overline{ab}$ and begin to radiate a wavelet. The disturbance due to the wave front $\overline{ab}$ stops at time t when the wavefront strikes point *d*. The semicircle in the figure represents wave-front of the wavelet at time *t*.

By referring to figure 3.3 for light wave propagation or using other methods, derive equation (1).

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ANSWER FORM
3

3A) Einstein's Mirror

Proof:

3B. Shift Frequency

Frequency Shift =

3C. Moving Mirror Equation

Proof: