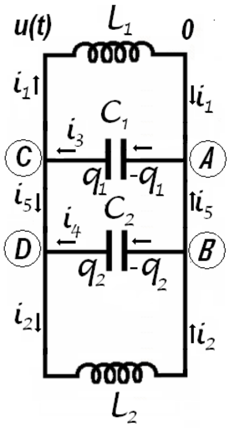
MARKING SCHEME FOR ANSWERS TO THE THEORETICAL QUESTION II

Part	MARKING SCHEME - THE THEORETICAL QUESTION II - DIFFERENT KIND OF OSCILLATION		Total Scores
II.a.	For: $\begin{cases} C = C_1 + C_2 \\ L = \dfrac{L_1 L_2}{L_1 + L_2} \end{cases}$	0.2p	2.5 points
	the impedance Z of the circuit $Z = \left\|\overline{Z}\right\| = \dfrac{1}{\sqrt{\left(\dfrac{1}{R}\right)^2 + \left(C \cdot \omega - \dfrac{1}{L \cdot \omega}\right)^2}} = \dfrac{1}{Y}$	0.2p	
	$i(t) = I \cdot \sqrt{2} \cdot \sin(\omega \cdot t)$	0.2p	
	$u(t) = U \cdot \sqrt{2} \cdot \sin(\omega \cdot t + \varphi)$	0.2p	
	$P = \dfrac{U^2}{R} = \dfrac{Z^2 \cdot I^2}{R}$	0.1p	
	the maximal active power is realized for the maximum value of the impedance that is the minimal value of the admittance $Y_{\min} = \dfrac{1}{R}$.	0.2p	
	$P_m = R \cdot I^2$	0.2p	
	$f_m = \dfrac{1}{2\pi \cdot \sqrt{C \cdot L}}$	0.2p	
	$P = \dfrac{1}{2} P_m$	0.1p	
	$\dfrac{Z^2 \cdot I^2}{R} = \dfrac{1}{2} R \cdot I^2$	0.1p	
	$\omega^2 \pm \dfrac{1}{R \cdot C}\omega - \dfrac{1}{L \cdot C} = 0$	0.2p	
	the pulsation of the current ensuring an active power at half of the maximum power $\begin{cases} f_+ = \dfrac{1}{2\pi}\left(\dfrac{1}{2}\sqrt{\left(\dfrac{1}{R \cdot C}\right)^2 + \dfrac{4}{L \cdot C}} + \dfrac{1}{2R \cdot C}\right) \\ f_- = \dfrac{1}{2\pi}\left(\dfrac{1}{2}\sqrt{\left(\dfrac{1}{R \cdot C}\right)^2 + \dfrac{4}{L \cdot C}} - \dfrac{1}{2R \cdot C}\right) \end{cases}$	0.2p	
	the bandwidth of the circuit $\Delta f = f_+ - f_- = \dfrac{1}{2\pi} \dfrac{1}{R \cdot C}$	0.1p	
	$\begin{cases} \dfrac{f_m}{\Delta f} = R\sqrt{\dfrac{C}{L}} \\ \dfrac{f_m}{\Delta f} = R\sqrt{\dfrac{(C_1 + C_2) \cdot (L_1 + L_2)}{L_1 \cdot L_2}} \end{cases}$	0.2p	
	final result: $\dfrac{f_m}{\Delta f} = 150$	0.1p	

II.b.	 For: $u(t) - L_1 \frac{di_1}{dt} = 0$ 0.1p $u(t) - L_2 \frac{di_2}{dt} = 0$ 0.1p $q_1 = C_1 \cdot u$ 0.1p $\frac{dq_1}{dt} = -i_3$ 0.1p $\frac{dq_1}{dt} = C_1 \cdot \frac{du}{dt}$ 0.1p $i_3 = -C_1 \cdot \frac{du}{dt}$ 0.1p $i_4 = -C_2 \cdot \frac{du}{dt}$ 0.1p $\begin{cases} \frac{di_1}{dt} = \frac{u}{L_1} \\ \frac{di_2}{dt} = \frac{u}{L_2} \end{cases}$ 0.2p $\begin{cases} \frac{di_3}{dt} = -C_1 \frac{d^2u}{dt^2} \\ \frac{di_4}{dt} = -C_2 \frac{d^2u}{dt^2} \end{cases}$ 0.2p the Kirchhoff rule of the currents for the point A $i_1 + i_5 = i_3$ 0.1p the Kirchhoff rule of the currents for the point B $i_4 + i_5 = i_2$ 0.1p $i_1 - i_3 = i_4 - i_2$ 0.1p $\frac{di_1}{dt} - \frac{di_3}{dt} = \frac{di_4}{dt} - \frac{di_2}{dt}$ 0.1p $\begin{cases} -\frac{u}{L_1} - \frac{u}{L_2} = C_1 \frac{d^2u}{dt^2} + C_2 \frac{d^2u}{dt^2} \\ -u \cdot \left(\frac{1}{L_1} + \frac{1}{L_2}\right) = \frac{d^2u}{dt^2} \cdot (C_1 + C_2) \end{cases}$ 0,2p $\begin{cases} -\frac{u}{L} = \frac{d^2u}{dt^2} \cdot C \\ \ddot{u} + \frac{1}{LC} u = 0 \end{cases}$ 0,2p $\omega = \frac{1}{\sqrt{L \cdot C}}$ 0,2p	2.2 points

	final result: $\omega = 10^5 \, rad \cdot s^{-1}$; $f = \dfrac{10^5}{2\pi} Hz$ 0.1p	
II.c.	For: $u(t) = A \cdot \sin(\omega \cdot t + \delta)$ 0.1p $\begin{cases} i_3 = -C_1 \dfrac{d}{dt}(A \cdot \sin(\omega \cdot t + \delta)) = -C_1 \cdot A \cdot \omega \cdot \cos(\omega \cdot t + \delta) \\ i_4 = -C_2 \dfrac{d}{dt}(A \cdot \sin(\omega \cdot t + \delta)) = -C_2 \cdot A \cdot \omega \cdot \cos(\omega \cdot t + \delta) \end{cases}$ 0.2p $\begin{cases} \dfrac{di_1}{dt} = \dfrac{u}{L_1} = \dfrac{1}{L_1} \cdot A \cdot \sin(\omega \cdot t + \delta) \\ \dfrac{di_2}{dt} = \dfrac{u}{L_2} = \dfrac{1}{L_2} \cdot A \cdot \sin(\omega \cdot t + \delta) \end{cases}$ 0.2p $\begin{cases} i_1 = \dfrac{1}{L_1 \cdot \omega} \cdot A \cdot \cos(\omega \cdot t + \delta) + M \\ i_2 = \dfrac{1}{L_2 \cdot \omega} \cdot A \cdot \cos(\omega \cdot t + \delta) + N \end{cases}$ 0.2p *Observation: in the expression above, A, M, N and δ are constants that must be determined using initially conditions ($u_0 = 40V$, $i_{01} = 0{,}1A$, $i_{02} = 0{,}2\,A$)* $\begin{cases} u(0) = A \cdot \sin(\delta) = u_0 \\ \sin(\delta) = \dfrac{u_0}{A} \end{cases}$ 0.1p $\begin{cases} i_{01} = \dfrac{1}{L_1 \cdot \omega} \cdot A \cdot \cos(\delta) + M \\ i_{02} = \dfrac{1}{L_2 \cdot \omega} \cdot A \cdot \cos(\delta) + N \end{cases}$ 0.2p $\begin{cases} i_1 - i_3 = i_4 - i_2 \\ \dfrac{1}{L_1 \cdot \omega} \cdot A \cdot \cos(\omega \cdot t + \delta) + M + C_1 \cdot A \cdot \omega \cdot \cos(\omega \cdot t + \delta) = \\ -C_2 \cdot A \cdot \omega \cdot \cos(\omega \cdot t + \delta) - \dfrac{1}{L_2 \cdot \omega} \cdot A \cdot \cos(\omega \cdot t + \delta) - N \end{cases}$ 0.2p *Observation: an identity as $A \cdot \cos\alpha + B \equiv C \cdot \cos\alpha + D$ is valuable for any value of the argument α only if $A = C$ and $B = D$* $\begin{cases} M + N = 0 \\ A \cdot \omega \cdot (C_1 + C_2) = -\dfrac{A}{\omega} \cdot \left(\dfrac{1}{L_1} + \dfrac{1}{L_2}\right) \end{cases}$ 0.1p $\begin{cases} i_{01} + i_{02} = A \cdot \cos(\delta) \cdot \dfrac{1}{\omega} \cdot \left(\dfrac{1}{L_1} + \dfrac{1}{L_1}\right) \\ \cos\delta = \dfrac{i_{01} + i_{02}}{A \cdot \dfrac{1}{\omega} \cdot \left(\dfrac{1}{L_1} + \dfrac{1}{L_1}\right)} \\ \cos\delta = \dfrac{(i_{01} + i_{02}) \cdot L \cdot \omega}{A} \end{cases}$ 0.2p	3.0 points

	$\begin{cases}(\cos(\delta))^2+(\sin(\delta))^2=1\\ \left(\frac{u_0}{A}\right)^2+\left(\frac{(i_{01}+i_{02})\cdot L\cdot\omega}{A}\right)^2=1\\ A=\sqrt{(u_0)^2+((i_{01}+i_{02})\cdot L\cdot\omega)^2}\end{cases}$ 0.1p the numerical value of the electrical tension on the jacks of the circuit $A=40\sqrt{26}V$ 0.1p $\begin{cases}\sin(\delta)=\frac{u_0}{A}\\ \sin(\delta)=\frac{1}{\sqrt{26}}\end{cases}$ 0.1p $\begin{cases}M=i_{01}-\frac{1}{L_1\cdot\omega}\cdot A\cdot\cos(\delta)\\ N=i_{02}-\frac{1}{L_2\cdot\omega}\cdot A\cdot\cos(\delta)\end{cases}$ 0.1p $\begin{cases}M=-0{,}1A\\ N=0{,}1A\end{cases}$ 0.1p $\begin{cases}i_1=\left(\frac{4\sqrt{26}}{100}\cdot\cos\left(10^5\cdot t+arctg(1/5)\right)-0{,}1\right)A=\tilde{i}_1-I_0\\ i_2=\left(\frac{2\sqrt{26}}{100}\cdot\cos\left(10^5\cdot t+arctg(1/5)\right)+0{,}1\right)A=\tilde{i}_2+I_0\end{cases}$ 0.2p *Observation: the currents through the coils are the superposition of sinusoidal currents having different amplitudes and a direct current passing only through the coils.* the direct current $I_0=0{,}1A$ 0.1p the alternative currents through the coils $\begin{cases}\tilde{i}_1=\left(\frac{4\sqrt{26}}{100}\cdot\cos\left(10^5\cdot t+arctg(1/5)\right)\right)A\\ \tilde{i}_2=\left(\frac{2\sqrt{26}}{100}\cdot\cos\left(10^5\cdot t+arctg(1/5)\right)\right)A\end{cases}$ 0.2p the currents through the capacitors $\begin{cases}i_3=\left(-\frac{4\sqrt{26}}{100}\cos\left(10^5\cdot t+arctg(1/5)\right)\right)A\\ i_4=\left(-\frac{2\sqrt{26}}{100}\cos\left(10^5\cdot t+arctg(1/5)\right)\right)A\end{cases}$ 0,2p $i_5=i_3-i_1$ 0.1p final result: $i_5=\left[-\frac{8\sqrt{26}}{100}\cos\left(10^5t+arctg\frac{1}{5}\right)+0{,}1\right]A$ 0.2p	
II.d.	For: the amplitude of the current through the inductance L_1 $\max(\tilde{i}_1)=\max\left(\frac{4\sqrt{26}}{100}\cdot\cos\left(10^5\cdot t+arctg(1/5)\right)A\right)$ 0.2p final result: $\max(\tilde{i}_1)\approx 0{,}2\ A$ 0.1p	0.3 points
Total score theoretical question III		***8.0 points***

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